Formulae And Tables

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FORMULAE

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i. Future Value of a lump sum (Single Flow)

$FV_n = PV(1 + i)^n$

Where,

FV_n	=	Future value of the initial flow n years hence
PV	=	Initial cash flow
i	=	Annual Rate of Interest
n	=	Life of investment

ii. Doubling Period $= 0.35 + \dfrac{69}{\text{Interest Rate}}$

iii. Future value of a lump sum with increased frequency of compounding

$FV_n = PV(1 + \frac{i}{m})^{m \times n}$

Where,

FV_n	=	Future value after 'n' years
PV	=	Cash flow today
i	=	Nominal Interest Rate per Annums
m	=	Number of times compounding is done during a year
n	=	Number of years for which compounding is done

iv. The relationship between Effective vs. Nominal Rate of Interest

$r = (1+\frac{i}{m})^m - 1$

Where,

r	=	Effective rate of interest
i	=	Nominal rate of interest
m	=	Frequency of compounding per year

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$$FVA_n = A\left[\frac{(1+i)^n - 1}{i}\right] = s_{\overline{n}|}$$

Where,

FVA_n	=	Accumulation at the end of n years
A	=	Amount deposited/invested at the end of every year for n years
i	=	Rate of interest (expressed in decimals)
n	=	Time horizon or number of installments
$s_{\overline{n}|}$	=	Accumulated value of an annuity

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Where,

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Where,

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Where,

i = Rate of interest

n = Number of years

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Where,

i = Rate of interest.

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v^n = Present value of the nth payment payable at the end of the nth year

= $1/(1 + i)^n$

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i = Rate of interest

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Where,

m = Deferment period

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i = Rate of interest

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$a_{\overline{n}|}$ = Present value of an Immediate Annuity Certain

n = The number of annuity installments

i = The rate of interest

v. Accumulated Value of an Annuity Due, $\ddot{s}_{\overline{n}|} = (1 + i)s_{\overline{n}|}$

Where,

$s_{\overline{n}|}$ = Present value of an Immediate Annuity Certain

n = The number of annuity installments

i = The rate of interest

vi. Present value of a deferred annuity due of Re. one p.a. for a term of n years certain and the deferment period is being m years

$= m|\ddot{a}_{\overline{n}|} = v^m \ddot{a}_{\overline{n}|}$

Where,

$v = \frac{1}{1+i}$

i = The rate of interest

$\ddot{a}_{\overline{n}|}$ = Present value of an Annuity due

vii. Accumulated value of a deferred annuity due of Re. one p.a. for a term of n years certain and the deferment period is being m years

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i = The rate of interest

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Where,

i = The rate of interest

$v = \frac{1}{1+i}$

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Where,

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$v = \frac{1}{1+i}$

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Where,

$\ddot{a}_{\overline{n|}}$ = The present value of an annuity due

$a_{\overline{n|}}$ = The present value of an annuity certain

n = Number of installments

i = The rate of interest

$v = \dfrac{1}{1+i}$

b. Present value of an increasing annuity due $(I\ddot{a})_{\overline{n|}} = \ddot{a}_{\overline{n|}} + \dfrac{\ddot{a}_{\overline{n|}} - nv^n}{i}$

Where,

$\ddot{a}_{\overline{n|}}$ = The present value of an annuity due

n = Number of installments

i = The rate of interest

$v = \dfrac{1}{1+i}$

c. Accumulated value of an increasing annuity due

$(I\ddot{s})_{\overline{n|}} = \ddot{s}_{\overline{n|}} + \dfrac{\ddot{s}_{\overline{n|}} - n \times (1+i)}{i}$

Where,

$\ddot{s}_{\overline{n|}}$ = The Accumulated value of an annuity due

n = Number of installments

i = The rate of interest

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Where,

i = The rate of interest

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Where,

d = The rate of discounting = $\dfrac{i}{1+i}$

i = The rate of interest

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Where,

A	=	The payment at the end of first year
D	=	The common difference
$a_{\overline{n}\|}$	=	The present value of an Annuity certain
n	=	The number of installments
v	=	$\frac{1}{1+i}$
i	=	The rate of interest

xvi. The Present Value of an Increasing Annuity wherein the consecutive periodical annuity payments are in a Geometric Progression

$$= A\left[\frac{1-R^n v^n}{(1+i)-R}\right]$$

Where,

v	=	$\frac{1}{1+i}$
R	=	The common multiple
i	=	The rate of interest
n	=	The number of installments
A	=	The amount of first installment

xvii. Accumulated Value of Increasing Immediate Annuity by Re. One per annum

$$= (Is)_{\overline{n}|} = s_{\overline{n}|} + \frac{s_{\overline{n}|} - n}{i}$$

Where,

$s_{\overline{n}\|}$	=	Accumulated value of an Annuity certain
n	=	Number of annuity installments
i	=	The rate of interest

xviii. The Accumulated Value of an Increasing Annuity wherein the consecutive periodical annuity payments are in an Arithmetic Progression

$$= A.\, s_{\overline{n}|} + D\left(\frac{s_{\overline{n}|} - n}{i}\right)$$

Where,

A	=	The amount of first installment
D	=	The amount of common difference
n	=	The number of installments
$s_{\overline{n}\|}$	=	The accumulated value of an annuity certain
i	=	The rate of interest

xix. The Accumulated Value of an Increasing Immediate Annuity wherein the consecutive periodical annuity payments are in a Geometric Progression

$$A\frac{(1+i)^n - R^n}{(1+i)-R}$$

Where,

A = The amount of first installment

R = The common ratio

i = The rate of interest

n = The number of installments

xx. Present Value of an Immediate Annuity of Re.1 p.a. for a term of n years under which payments are made p times a year

$$a_{\overline{n}|}^{(p)} = a_{\overline{n}|} \times \frac{i}{i^{(p)}}$$

Where,

i = The rate of interest per annum

$a_{\overline{n}|}$ = The present value of an Annuity certain

$i^{(p)}$ = $\left[(1+i)^p - 1\right] \times p$

xxi. Accumulated Value of an Immediate Annuity of Re.1 p.a. for a term of n years under which payments are made p times a year

$$s_{\overline{n}|}^{(p)} = s_{\overline{n}|}\left(\frac{i}{i^{(p)}}\right) \times v$$

Where,

i = The rate of interest per annum

v = $\frac{1}{1+i}$

$s_{\overline{n}|}$ = The Accumulated value of an Annuity certain

$i^{(p)}$ = $\left[(1+i)^p - 1\right] \times p$

xxii. Present Value of an Annuity Due of Re. 1 p.a. for n years under which payments are made 'p' times a year

$$\ddot{a}_{\overline{n}|}^{(p)} = a_{\overline{n}|}\left(\frac{i}{i^{(p)}} + \frac{i}{p}\right)$$

Where,

i = The rate of interest per annum

$a_{\overline{n}|}$ = The present value of an Annuity certain

$i^{(p)}$ = $\left[(1+i)^p - 1\right] \times p$

xxiii. Accumulated Value of an Annuity Due of Re.1 p.a. for n years under which payments are made 'p' times a year

$$\ddot{s}_{\overline{n}|}^{(p)} = s_{\overline{n}|}\left(\frac{i}{i^{(p)}} + \frac{i}{p}\right)$$

Where,

i = The rate of interest per annum

$s_{\overline{n}|}$ = The accumulated value of an Annuity certain

$i^{(p)}$ = $\left[(1+i)^{p} - 1\right] \times p$

xxiv. An immediate annuity for n years where payment of 'r' are made at each interval of 'r' years, n being an exact multiple of 'r' and the number of payments being $\frac{n}{r}$

a. Present value of the above Annuity $= a_{\overline{n}|}^{(1/r)} = \frac{r a_{\overline{n}|}}{s_{\overline{r}|}}$

Where,

$a_{\overline{n}|}$ = The present value of an Annuity certain for n years

$s_{\overline{r}|}$ = The accumulated value of an Annuity for r years

b. Accumulated value of the above annuity $s_{\overline{n}|}^{(1/r)} = \frac{r s_{\overline{n}|}}{s_{\overline{r}|}}$

Where,

$s_{\overline{n}|}$ = The present value of an Annuity certain for n years

$s_{\overline{r}|}$ = The accumulated value of an Annuity for r years

xxv. Present value and accumulated value of an annuity due for n years where payments of 'r' are made at interval of 'r' years, n being exact multiple of 'r'

a. Present value: $\ddot{a}_{\overline{n}|}^{(1/r)} = r\frac{a_{\overline{n}|}}{a_{\overline{r}|}}$

Where,

$a_{\overline{n}|}$ = The present value of an Annuity certain for n years

$a_{\overline{r}|}$ = The present value of an Annuity for r years

b. Accumulated value: $\ddot{s}_{\overline{n}|}^{(1/r)} = r\frac{s_{\overline{n}|}}{a_{\overline{r}|}}$

Where,

$s_{\overline{n}|}$ = The present value of an Annuity certain for n years

$a_{\overline{r}|}$ = The present value of an Annuity for r years

xxvi. Capital Redemption Policies

a. The Amount of Annual Premium

$$P_{\overline{n}|} = \frac{1}{s_{\overline{n+1}|} - 1}$$

Where,

$s_{\overline{n+1}|}$ = The Accumulated value of an Annuity certain for a period of n + 1 years at a rate of interest of i per annum

b. Single Premium $A_{\overline{n}|} = \frac{1}{(1+i)^n} = v^n$

Where,

i = The rate of interest per annum

n = The number of years

xxvii. Average Interest Yield on the Life Fund $= \frac{2I}{A+B-I}$

Where,

A = The fund at the beginning

B = The fund at the end of the year

I = The interest earned during the year after payment of tax

xxviii. Office premium $A'_n = A_n + 1A'_n$

Where,

A_n = Pure premium

l = Premium loading factor.

3. Demography

i. Crude Death Rate $= \frac{D}{P} x1000$

Where,

D stands for total number of deaths in a given year, and

P stands for the size of the mid year population.

ii. Fertility Rates:

a. Crude Fertility Rate (CFR) $= \frac{\text{Number of births during a specified period}}{\text{Total number of mid-year population of women}}$

b. General Fertility Rate (GFR) $= \frac{\text{Number of births during a specified period}}{\text{Total number of mid-year population of women aged between 15-49}}$

c. Age-Specific Fertility Rate, at age y ($ASFR_y$) $= \frac{\text{Number of births in a specified period to women aged y years}}{\text{Total number of mid-year population of women aged y years}}$

iii. Marriage Rates:

a. Crude Marriage Rate (CMR) $= \frac{\text{Number of marriages taken place during a specific period}}{\text{Total number of mid-year population}}$

b. General Marriage Rate (GMR) $= \frac{M}{P_{15+}}$ x 1000

Where,

M stands for the total number of marriages solemnized in a given period and

P_{15+} stands for the mid-year population of age 15 years or more

c. Age-Specific Marriage Rate at age y ($ASMR_y$) = $\frac{\text{Number of people married at age y during the year}}{\text{Total number of mid-year population at age y}}$

iv. Migration Rate of any area r = $\frac{\text{Number of people moving in and out of area r in a specified period}}{\text{Total number of population in area r at the beginning of the time period}}$

v. Dependency ratio = $\frac{\text{Economically inactive population}}{\text{Economically active population}}$

4. Survival Models

i. The estimated probability of deaths in an interval computed per unit time,

$$F_i = \frac{P_i - P_{i+1}}{h_i}$$

Where,

F_i = Respective probability density in the ith interval

P_i = Estimated cumulative proportion surviving at the beginning of the ith interval (at the end of the interval i – 1)

P_{i+1} = Cumulative proportion surviving at the end of the ith interval

h_i = Width of the ith interval

ii. Exponential Distribution

$$F(T) = \lambda e^{-\lambda T} = \frac{1}{m} e^{(-1/m)T}$$

Where,

λ = Constant death rate in terms of deaths per unit of measurement

m = Mean time between deaths

T = Operating time, Life or age in hours, cycles, etc.

iii. Weibull Distribution

$$f(T) = \frac{\beta}{\eta}\left(\frac{T}{\eta}\right)^{\beta-1} e^{-(T/\eta)^{\beta}}$$

Where,

$f(T) \geq 0, T \geq 0, \beta \geq 0$ and $\eta > 0$

η = Scale parameter

β = Shape parameter (or slope).

5. Mortality Tables

i. The probability that a person of age x years dies within one year

$$\therefore q_x = \frac{\text{Number of deaths between age x and x+1}}{\text{Total number of persons living at age x}} = \frac{d_x}{l_x} = \frac{l_x - l_{x+1}}{l_x}$$

ii. The probability that a person of age x years survives another one year

$$\therefore p_x = \frac{\text{Number of survivors to age } (x+1)}{\text{Total number of persons living at age x}} = \frac{l_{x+1}}{l_x}$$

iii. Expectation of life at age x is given by:

$$\therefore e_x = \frac{N'_{x+1}}{l_x}$$

Where,

$$N'_{x+1} = \sum_{t=x+1}^{w-1} l_t$$

w = Terminal age

iv. Central Death Rate: $m_x = \dfrac{2q_x}{2-q_x}$

Where,

q_x = The probability that a person of age x years dies within one year

v. The probability that a person of age x years survives another n years

$${}_np_x = \frac{\text{No. of persons living at age } x+n}{\text{No. of persons living at age x}} = \frac{l_{x+n}}{l_x}$$

vi. The probability that a person of age x years dies within the next n years

$$|_m q_x = \frac{\text{Total no. of persons dying between ages x and } x+m}{\text{Total no. of persons living at age x}} = \frac{l_x - l_{x+m}}{l_x}$$

vii. The probability that a person of age x years will die within n years following m years from now ${}_m|_nq_x = \dfrac{\text{No. of deaths between ages } x+m \text{ and } x+m+n}{\text{No. of persons living at age x}}$

$$= \frac{l_{x+m} - l_{x+m+n}}{l_x}$$

6. Assurance and Annuity Benefits

i. The present value of a term assurance of Re.1.00 payable on death during an n year period is given by

$$A^1_{x:\overline{n|}} = \frac{1}{l_x}(vd_x + v^2d_{x+1} + v^3d_{x+2} + \ldots.. + v^nd_{x+n-1})$$

Where,

x = Age of the person

n = Number of years the policy is in force

i = The rate of interest per annum

v = $\dfrac{1}{1+i}$

d_x = The number of deaths between age x and x + 1

l_x = Total number of persons living at age x

ii. The present value of benefit of Re.1.00 payable to an insured against a pure endowment policy for n years taken at an age x is given by:

$$A_{x:\overset{1}{\overline{n|}}} = v^n \times \frac{l_{x+n}}{l_x}$$

Where,

x	=	Age of the person
n	=	Number of years the policy is in force
i	=	The rate of interest per annum
v	=	$\frac{1}{1+i}$
l_x	=	Total number of persons living at age x
l_{x+n}	=	Total number of persons living at age x + n

iii. The present value of benefit of Re.1.00 payable to an insured against an endowment assurance policy for n years taken at an age x is given by:

$$A_{x:\overline{n|}} = A^1_{x:\overline{n|}} + A_{x:\overset{1}{\overline{n|}}}$$

Where,

$A^1_{x:\overline{n|}}$	=	The present value of benefit in a Term Insurance Policy
$A_{x:\overset{1}{\overline{n|}}}$	=	The present value of benefit in a Pure Endowment Policy

iv. The present value of an increasing whole life assurance on the life of a person aged x at entry where the sum assured is Re.1.00 in the first year, Rs.2.00 in the second year, Rs.3.00 in the third year and so on, is given by:

$$(IA)_x = \frac{1}{l_x}(vd_x + 2v^2d_{x+1} + 3v^3d_{x+2} + 4v^4d_{x+3} + ...)$$

Where,

x	=	Age of the person
n	=	Number of years the policy is in force
i	=	The rate of interest per annum
v	=	$\frac{1}{1+i}$
d_x	=	The number of deaths between age x and x + 1
l_x	=	Total number of persons living at age x

v. Commutation Functions:

a. $D_x = v^x l_x$

b. $C_x = v^{x+1} d_x$

c. $M_x = C_x + C_{x+1} + C_{x+2} + \ldots$

d. $R_x = M_x + R_{x+1}$

vi. Present value of the assurance benefits to the insured in terms of the commutation functions are as follows:

a. Temporary Assurance Policy, $A^1_{x:\overline{n|}} = \frac{M_x - M_{x+n}}{D_x}$

b. Whole Life Assurance Policy, $A_x = \frac{1}{D_x}(M_x)$

c. Pure Endowment Assurance Policy, $A_{x:\overline{n}|}^{\;\;1} = \frac{D_{x+n}}{D_x}$

d. Endowment Assurance Policy, $A_{x:\overline{n}|} = \frac{M_x - M_{x+n} + D_{x+n}}{D_x}$

e. Double Endowment Assurance Policy:

$$DA_x = \frac{M_x - M_{x+n} + 2D_{x+n}}{D_x}$$

f. Increasing Temporary Assurance Policy:

$$(IA)^1_{x:\overline{n}|} = \frac{R_x - R_{x+n} - nM_{x+n}}{D_x}$$

g. Increasing Whole Life Assurance Policy, $(IA)_x = \frac{R_x}{D_x}$

h. Special Endowment Assurance Policy that provides increasing death benefit and increasing survival benefits:

$$(IA)_{x:\overline{n}|} = \frac{R_x - R_{x+n} - nM_{x+n} + nD_{x+n}}{D_x}$$

i. Deferred Temporary Assurance Policy:

$$t|A^1_{x:\overline{n}|} = A^{\;1}_{x:\overline{t+n}|} - A^1_{x:\overline{t}|}$$

j. Deferred Whole Life Assurance Policy, $t|A_x = A_x - A^{\;1}_{x:\overline{t}|}$

vii. Present value of an immediate annuity for life of Re.1.00 to an annuitant of age x years is given by $a_x = \frac{N_{x+1}}{D_x}$

viii. Present value of an immediate annuity due for life of Re.1.00 to an annuitant of age x years is given by $\ddot{a}_x = 1 + a_x$

Where,

$$a_x = \frac{N_{x+1}}{D_x}$$

ix. Present value of a deferred life annuity for Re.1.00 to an annuitant of age x years for a deferment period of t years is given by

$$t|a_x = \frac{N_{x+t+1}}{D_x}$$

x. Present value of a deferred life annuity for Re.1.00 due to an annuitant of age x years for a deferment period of t years is given by:

$$t|\ddot{a}_x = \frac{N_{x+t}}{D_x}$$

xi. Present value of a temporary immediate life annuity for life of Re.1.00 to an annuitant of age x years for a term of n years is given by

$$a_{x:\overline{n}|} = \frac{N_{x+1} - N_{x+n+1}}{D_x}$$

xii. Present value of a deferred temporary immediate life annuity for life of Re.1.00 to an annuitant of age x years for a term of n years to be started after a deferment period of t years is given by

$$t|\ddot{a}_{x:\overline{n}|} = a_{x:\overline{n+t-1}|} - a_{x:\overline{t-1}|}$$

xiii. Present value of an increasing life annuity in terms of commutation function S_x is given by:

$$(I\ddot{a})_x = \frac{S_x}{D_x}$$

xiv. Present value of an increasing life annuity in terms of commutation functions is given by:

$$(Ia)_{x:\overline{n}|} = \frac{1}{D_x}[S_x - S_{x+n} - nN_{x+n}]$$

xv. Present value of a life annuity with m number of payments in a year is given by:

$$a^{(m)}_{x:\overline{n}|} = a_{x:\overline{n}|} + \frac{m+1}{2m}\left(1 - \frac{D_{x+n}}{D_x}\right)$$

7. Premiums for Assurance and Annuity Plans

i. The amount of level annual premium to be paid by a person of age x at the beginning of each year to have a term assurance plan for n years:

$$P^1_{x:\overline{n}|} = \frac{M_x - M_{x+n}}{N_x - N_{x+n}}$$

ii. The amount of level annual premium to be paid by a person of age x at the beginning of each year to have a pure endowment assurance plan for n years:

$$P_{x:\overline{n}|}^{\;\;1} = \frac{D_{x+n}}{N_x - N_{x+n}}$$

iii. The amount of level annual premium to be paid by a person of age x at the beginning of each year to have an endowment assurance plan for n years:

$$P_{x:\overline{n}|} = \frac{M_x - M_{x+n} + D_{x+n}}{N_x - N_{x+n}} = P^1_{x:\overline{n}|} + P_{x:\overline{n}|}^{\;\;1}$$

iv. The amount of level annual premium to be paid by a person of age x at the beginning of each year to have a whole life assurance plan:

$$P_x = \frac{M_x}{N_x}$$

v. The amount of level annual premium to be paid by a person of age x at the beginning of each year to have a limited payment assurance plan for a limited period of t years:

$$t^{P_x} = \frac{M_x}{N_x - N_{x+t}}$$

vi. If the payment of premiums is limited to a shorter period 't' where t < n years in an endowment assurance plan then the level premium is denoted by $t^{P_{x:\overline{n}|}}$

$$t^{P_{x:\overline{n}|}} = \frac{M_x - M_{x+n} + D_{x+t}}{N_x - N_{x+t}}$$

vii. The present value of a decreasing term assurance policy

$$A = \frac{S(nM_x - R_{x+1} + R_{x+n+1})}{D_x}$$

Where,

S_n = The amount of sum assured in the first year

S = The amount by which the amount of sum assured decreases in every year

R_x, M_x and D_x are the different communication functions.

If 'P' is the net annual premium limited for a fixed period 't' years

Where $t \le \frac{2n}{3}$, then $P = \frac{S(nM_x - R_{x+1} - R_{x+n+1})}{N_x - N_{x+t}}$

Where,

S = The amount by which, the amount of sum assured is reduced every year

viii. Children's Deferred Assurances:

a. Annual premium for the Children's Deferred Whole Life Assurance plan is given by:

$$\frac{v^{21-x} \times A_{21}}{\ddot{a}_{\overline{21-x}|} + v^{21-x} \times \ddot{a}_{21}}$$

Where, × is the age of the child.

b. Annual premium for the Children's Deferred Endowment Assurance plan, maturing at an age m is given by:

$$\frac{v^{21-x} A_{\overline{21:m-21}|}}{\ddot{a}_{\overline{21-x}|} + v^{21-x} \; a_{\overline{21:m-21}|}}$$

c. Additional Annual premium payable during the deferment period to get the premium waiver benefit in the event of death of the father during the deferment period, corresponding to the basic annual premium of Rs.100, given by:

$$= \frac{100\left(\ddot{a}_{\overline{21-x}|}\right)}{\ddot{a}_{\overline{y:21-x}|}}$$

Where,

y = The age of the father on the date of commencement of the policy.

ix. Net single premium for an immediate annuity of Re.1.00 per annum payable in arrear every year for n years certain and thereafter during the life time of the annuitant of age x at entry is given by:

$$a_{\overline{n}|} + \frac{D_{x+n}}{D_x} \text{ x } (a_{x+n})$$

x. Net annual premium P payable for t years for the deferred annuity of Re.1 per annum payable m times in a year for n years certain and thereafter during the lifetime of the annuitant of age x at entry with a deferment period of t-years is given by:

$$P = \frac{a^{(m)}_{\overline{n}|} + \frac{D_{x+n+t}}{D_{x+t}}\left(a_{x+n+t} + \frac{m-1}{2m}\right)}{\ddot{s}_{\overline{t}|}}$$

xi. Calculation of premiums when frequency of payment is m times a year:

a. Let $P_x^{(m)}$ represents the net premium per annum payable for a whole life assurance at the end of the year of death of (x). A premium of $\frac{1}{m}P_x^{(m)}$ is payable at the commencement of each m th period of a year which (x) enters.

$$P_x^{(m)}\ \ddot{a}_x^{(m)} = \frac{P_x}{1-\left(\frac{m-1}{2m}\right)(P_x + d)}$$

Where,

P_x = The amount of annual premium

$\ddot{a}_x^{(m)}$ = The present value of annuity due where the premiums are paid m times a year

d = Discount factor = $\frac{i}{1+i}$

i = The rate of interest per annum

b. For an endowment assurance Re.1 on (x) for a term of n years for which premiums are payable m times, we have,

$$P_{x:\overline{n}|}^{(m)} = \frac{P_{x:\overline{n}|}}{1-\frac{m-1}{2m}\left\{P_{x:\overline{n}|}^{1} + d\right\}}$$

Where,

$P_{x:\overline{n}|}$ = Level annual premium

c. For whole life limited payment policy we have,

$$t^{Px(m)} = \frac{t^{Px}}{1-\frac{m-1}{2m}\left\{P_{x:\overline{t}|}^{1} + d\right\}}$$

Where,

t^{Px} = Level annual premium

d. For limited payment endowment policy :

$$P_{x:\overline{n}|}^{(m)} = \frac{t^{Px:\overline{n}|}}{1-\frac{m-1}{2m}\left(P_{x:\overline{t}|}^{1} + d\right)}$$

Where,

$t^{P_{x:\overline{n}|}}$ = Level annual premium

xii. Premiums for additional risks:

a. The sum assured is subject to an initial debt of tD that reduces by D every year. The additional premium payable for an whole life assurance policy will be: $P'_x - P_x = \frac{D(tM'_x - R'_{x+1} + R'_{x+t+1})}{N'_x}$

Where, M'_x, R' and N'_x are the commutation functions for additional risks.

b. The sum assured is subject to an initial debt of tD that reduces by D every year. The additional premium payable for an endowment assurance policy will be:

$$P'_{x:\overline{n|}} - P_{x:\overline{n|}} = \frac{D(tM'_x - R'_{x+1} + R'_{x+t+1})}{N'_x + N'_{x+n}}$$

xiii. Calculation of Office premium:

a. Whole life assurance policy:

$$P^1 = \frac{S\left[P_x + \frac{(I_2 - K_2)}{\ddot{a}_x} + K_2\right]}{1 - \frac{(I_1 - K_1)}{\ddot{a}_x} - K_1}$$

Where,

P^1	=	Office premium
P_x	=	Level annual premium
$\ddot{a}_x$	=	The Present value of an immediate annuity due for life of Re.1.00 to an annuitant of age x years
I_1 and I_2	=	Initial expenses which are expressed per unit of premium and per unit of sum assured respectively
K_1 and K_2	=	Renewel expenses equal to which are expressed per unit of premium and per unit of sum assumed respectively

b. Endowment Assurance Policy: $P^1 = \frac{S\left[P_{x:\overline{n|}} + \frac{(I_2 - K_2)}{\ddot{a}_{x:\overline{n|}}} + K_2\right]}{1 - \frac{(I_1 - K_1)}{\ddot{a}_{x:\overline{n|}}} - K_1}$

Where,

P^1	=	Office premium
$P_{x:\overline{n|}}$	=	Level annual premium
$\ddot{a}_{x:\overline{n|}}$	=	The Present value of an immediate annuity due for life of Re.1.00 to an annuitant of age x years for a term of n years
I_1 and I_2	=	Initial expenses which are expressed per unit of premium and per unit of sum assured respectively
K_1 and K_2	=	Renewel expenses equal to which are expressed per unit of premium and per unit of sum assumed respectively.

8. Credibility Theory

i. When Normal approximation is applied to the Poisson distribution then, the probability (P) that observation X is within ± k of the mean μ is given by: $P = 2\,\Phi(k\sqrt{n}) - 1$

Where,

n = Number of claims

Φ stands for normal distribution

ii. The standard for full credibility for severity is given by

$N = n_0 CV_s^2$

Where,

n_0 = The full credibility standard for frequency

CV_s^2 = The coefficient of variation for the claim size distribution

iii. Process variance for pure premium is given by:

Var (PP) = $\mu_f \sigma_S^2 + \mu_S^2 \sigma_f^2$

Where,

μ_f = Mean of the claim frequency distribution

μ_s = Mean of the claim severity distribution

σ_f^2 = Variance of the claim frequency distribution

σ_S^2 = Variance of the claim severity distribution

iv. The expected number required for full credibility of pure premium

$n_F = n_0(1 + CV_S^2)$

Where,

n_0 = The ratio between the mean pure premium and the standard deviation of pure premiums

CV_S = The coefficient of severity

v. If the Poisson assumption does not hold good, general formula for the standard for full credibility is given by:

$n_F = \{y^2/k^2\}(\sigma_f^2/\mu_f + \sigma_S^2/\mu_s^2)$

Where,

k = Allowance for the variance of the observed sampled frequency rate

y = Standard normal variation

μ_f = Mean of the claim frequency distribution

μ_s = Mean of the claim severity distribution

σ_f^2 = Variance of the claim frequency distribution

σ_S^2 = Variance of the claim severity distribution

vi. B Ü HLAMANN Credibility is given by

$$Z = \frac{N}{N+k}$$

Where,

N is the number of observations and k is the B ü hlamann credibility parameter.

9. Loss Distributions and Risk Models

i. Poisson Distribution:

a. $P(N = r) = \frac{e^{-n} n^r}{r!}$ r = 0, 1, 2,............

b. Mean = n

c. Variance = n

ii. Lognormal Distribution:

a. The PDF is defined as:

$$f(x) = \frac{1}{\sigma x \sqrt{2\pi}} \times \exp\left[\frac{-1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right] \quad x > 0$$

b. Mean = $\exp\left(\mu + \frac{1}{2}\sigma^2\right)$

c. Variance = $\exp(2\mu + \sigma^2)\,[\exp(\sigma^2) - 1]$

iii. Pareto Distribution:

a. The PDF is defined as: f(x)

$$= \frac{\alpha}{\beta}\left(\frac{\beta}{\beta + x}\right)^{\alpha+1}, \quad x > 0$$

b. Mean of a pareto distribution is given by,

$$E(X) = \frac{\beta}{\alpha - 1}$$

c. $\text{Var}(X) = \frac{\alpha\beta^2}{(\alpha-2)(\alpha-1)^2}$

iv. Gamma Distribution:

a. The PDF is defined as: $f(x) = \frac{\beta^\alpha}{\tau(\alpha)} e^{-\beta x} (x)^{\alpha-1} \quad 0 \le x < \infty$

b. Mean = $\frac{\alpha}{\beta}$

c. Variance = $\frac{\alpha}{\beta^2}$

v. Individual Risk Model:

a. Expected Aggregate Loss: E(S)

$$= \sum_{j=1}^{n} E(Y_j) = \sum_{j=1}^{n} q_j \mu_j$$

Where,

Y_j = The amount of claim from the n-th policy

q_j = The probability of a claim from the j-th policy

μ_j = The amount of benefit associated with the j-th policy

b. Variance of aggregate loss: Var (S)

$$= \sum_{j=1}^{n} \text{Var}(Y_j) = \sum_{j=1}^{n} q_j (1 - q_j) \mu_j^2$$

Where,

Y_j = The amount of claim from the n-th policy

q_j = The probability of a claim from the j-th policy

μ_j = The amount of benefit associated with the j-th policy

vi. Collective Risk Model:

a. Mean: $E(S) = E[Y_i]\,E[n]$

b. Variance $\text{Var}(S) = E[n]\,\text{Var}[Y_i] + E[Y_i]\,\text{Var}[n]$

Where,

Y_i = Amount of claim from the i-th policy

n = Number of policies.

10. Policy Values

i. For a Whole Life assurance policy, policy value is given by:

${}_tV_x = A_{x+t,} P_x$

Where,

A_{x+t}	=	Present value of Assurance benefits
P_x	=	Level annual premium

ii. The policy value under prospective method for an Endowment assurance policy is given by:

$${}_tV_{x:\overline{n}|} = A_{x+t:\overline{n-t}|} - P_{x:\overline{n}|} \,.\, \ddot{a}_{x+t:\overline{n-t}|}$$

Where,

$A_{x+t:\overline{n-t}|}$	=	Present value of Assurance benefits an age of x + t years
$P_{x:\overline{n}|}$	=	Level Annual Premium
$\ddot{a}_{x+t:\overline{n-t}|}$	=	Present value of an immediate annuity due for life of Re.1.00 to an annuitant of age x + t years for a term of n – t years.

iii. Under prospective method, the policy value for Temporary assurance policy is given by:

$${}_tV^1_{x:\overline{n}|} = A^1_{x+t:\overline{n-t}|} - P_{x+t:\overline{n-t}|} \,.\, \ddot{a}_{x+t:\overline{n-t}|}$$

Where,

$A^1_{x+t:\overline{n-t}|}$	=	Present value of Assurance benefits an age of x + t years
$P_{x+t:\overline{n-t}|}$	=	Level Annual Premium
$\ddot{a}_{x+t:\overline{n-t}|}$	=	Present value of an immediate annuity due for life for Re.1.00 to an annuitant of age x + t years for a term of n – t years.

11. Surplus and it's Distribution

i. Loading profit that is profit due to lower expenses is expressed as:

$$(P' - P - E) \text{ x } \left(1+\frac{i}{2}\right)$$

Where,

P'	=	Total amount of office premium received
P	=	Total of premiums taken credit for in the last valuation
E	=	Actual expenses
i	=	Valuation rate.

II. Economics

1. Supply and Demand Analysis

i. Price elasticity of demand

a. Point Elasticity

$$e_p = \frac{\partial Q}{\partial P} \times \frac{P}{Q}$$

Where,

∂Q = Infinitisimal change in quantity demanded

∂P = Infinitisimal change in price

P = Original price of the good

Q = Original quantity demanded of the good

b. Arc Elasticity

$$e_p = \frac{\Delta Q}{\Delta P} \times \frac{P_0 + P_1}{Q_0 + Q_1}$$

Where,

ΔQ = Change in quantity demanded

ΔP = Change in price of the good

P_0 = Original price of the good

P_1 = New price of the good

Q_0 = Original quantity demanded of the good

Q_1 = New quantity demanded of the good

ii. Marginal Revenue

$$MR = AR\left\{1 - \frac{1}{|e_p|}\right\}$$

Where,

AR = Average revenue

e_p = Price elasticity of demand

iii. Income elasticity of demand

$$e_y = \frac{\partial Q}{\partial Y} \times \frac{Y}{Q}$$

Where,

∂Q = Change in quantity demanded

∂Y = Change in income of the consumer

Y = Income of the consumer

Q = Quantity demanded of the good

iv. Cross price elasticity of demand

$$e_{cij} = \frac{\partial Q_i}{\partial P_j} \times \frac{P_j}{Q_i}$$

Where,

∂Q_i = Change in quantity demanded of the good i

∂P_j = Change in price of the good j

P_j = Price of the good j

Q_i = Quantity demanded of the good i

v. Promotional elasticity of demand

$$e_A = \frac{\partial Q}{\partial A} \times \frac{A}{Q}$$

Where,

∂Q = Change in quantity demanded

∂A = Change in units of advertisement expenditure on the good

A = Units of advertisement expenditure on the good

Q = Quantity demanded of the good

vi. Price-elasticity of supply

$$e_s = \frac{\partial Q_s}{\partial P} \times \frac{P}{Q_s}$$

Where,

P = Price of the good

Q_s = Quantity supplied of the good

∂Q_s = Change in quantity supplied on the good

∂P = Change in price of the good.

2. Consumer Behavior and Analysis

i. Marginal Rate of Substitution of good X for good Y

$$MRS_{X,Y} = \frac{MU_X}{MU_Y}$$

Where,

MU_X = Marginal Utility of good X

MU_Y = Marginal Utility of good Y

ii. Consumer equilibrium

$$\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y}$$

Where,

MU_X = Marginal Utility of good X

MU_Y = Marginal Utility of good Y

P_X = Price of good X

P_Y = Price of good Y

iii. Budget constraint

$$I = P_X X + P_Y Y$$

Where,

I = Income of the consumer

X = Number of units of good X

Y = Number of units of good Y

P_X = Price of good X

P_Y = Price of good Y.

3. Production Analysis

i. Average product of labor

$$AP_L = \frac{TP_L}{L}$$

Where,

TP_L = Total product of labor

L = Number of labor units

ii. Marginal product of labor

$$MP_L = \frac{\Delta TP_L}{\Delta L}$$

Where,

ΔTP_L = Change in total product of labor

ΔL = Change in the number of labor units

iii. Marginal rate of technical substitution between Labor (L) and Capital (K)

$$MRTS_{L,K} = \frac{MP_L}{MP_K}$$

Where,

MP_L = Marginal product of labor

MP_K = Marginal product of capital

iv. Cost constraint of a firm

$C_0 = wL + rK$

Where,

C_0 = A given amount of money that the firm spends

L = Number of labor units

K = Number of capital units

w = Wage rate

r = Interest rate

v. Efficient input combination

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$

Where,

MP_L = Marginal product of labor

MP_K = Marginal product of capital

w = Wage rate

r = Interest rate.

4. Analysis of Costs

i. $TC = TFC + TVC$

Where,

TC = Total cost

TFC = Total fixed cost

TVC = Total variable cost

ii. $AFC = \frac{TFC}{Q}$

Where,

TFC = Total fixed cost

Q = Number of units produced

iii. $MC = \frac{\partial TC}{\partial Q}$

Where,

∂TC = Change in total cost

∂Q = Change in quantity produced

iv. Break-even output (Q) $= \frac{FC}{P - AVC}$

Where,

P = Price

FC = Fixed cost

AVC = Average variable cost.

5. Market Structure: Perfect Competition

i. Profit of a firm (π) = TR – TC

Where,

TR = Total revenue

TC = Total cost

ii. Tax burden on the buyer $= \frac{e_s}{e_d + e_s} \times \text{Tax}$

Where,

e_s = Price elasticity of supply

e_d = Price elasticity of demand

iii. Profit maximization

a. First order condition

MC = MR = AR = P

b. Second order condition

$$\frac{\partial^2 TR}{\partial Q^2} < \frac{\partial^2 TC}{\partial Q^2}$$

Where,

TR = Total revenue

TC = Total cost

MR = Marginal revenue

MC = Marginal cost

AR = Average revenue

P = Price

Q = Quantity.

6. Market Structure: Monopoly

i. Profit maximization

Marginal Revenue (MR) = Marginal Cost (MC)

ii. Lerner Index of monopoly power

a. $L = \frac{P - MC}{P}$

b. $\frac{P - MC}{P} = \frac{1}{|e_p|}$

Where,

P = Price

MC = Marginal Cost

e_p = Elasticity of demand

iii. The Herfindahl's Index (H)

$H = S_1^2 + S_2^2 + S_3^2 + ...S_n^2$

Where,

S_1 = % share of the largest firm in the market

S_2 = % share of the second largest firm in the market

S_n = % share of the nth firm in the market.

7. Market Structure: Oligopoly

i. Output determination

$Q_n = Q_P \left[\frac{n}{n+1}\right]$

Where,

Q_p = Output if the market would be a competitive one

n = Number of firms in Oligopoly.

8. Measurement of Macro Economic Aggregates

i. Gross = Net + Depreciation

ii. Market Price = Factor Cost + [Indirect Tax – Subsidy]

iii. National = Domestic + Net Factor Income from Abroad

iv. The Laspeyre Price Index

$$I_t = \frac{\sum_{i=1}^{n} P_i^t q_i^0}{\sum_{i=1}^{n} P_i^0 q_i^0} \times 100$$

Where,

q_i^0 = Quantity of ith good purchased in the base year

p_i^0 = Price of the ith good in the base year

q_i^t = Quantity of ith good purchased in the current year

p_i^t = Price of the ith good in the current year

v. GNP Deflator = $\frac{\text{Nominal GNP}}{\text{Real GNP}}$.

9. The Simple Keynesian Model of Income Determination

i. $Y = C + I + G + E - M$

Where,

Y = Equilibrium income

C = Consumption expenditure

I = Investment expenditure

G = Government expenditure

E = Exports

M = Imports

ii. Average Propensity to Consume (APC) $= \frac{C}{Y}$

Where,

C = Consumption expenditure

Y = Income

iii. Marginal Propensity to Consume (MPC) $= \frac{\Delta C}{\Delta Y}$

Where,

ΔC = Change in consumption expenditure

ΔY = Change in income

iv. Average Propensity to Save (APS) $= \frac{S}{Y}$

Where,

S = Savings

Y = Income

v. Marginal Propensity to Save (MPS) $= \frac{\Delta S}{\Delta Y}$

Where,

ΔS = Change in savings

ΔY = Change in income

vi. Multiplier (m) $= \frac{1}{[1-\beta(1-t)-\pi+\mu]}$

Where,

β = Marginal propensity to consume

t = Tax coefficient

π = Induced investment coefficient

μ = Marginal propensity to import.

10. Income Determination Model including Money and Interest

i. Goods market equilibrium

$Y = C + I + G + X - M$

Where,

C = Consumption expenditure

Y = Income

I = Investment expenditure

G = Government expenditure

E = Exports

M = Imports

ii. Money market equilibrium

$M_s = M_d$

Where,

M_s = Supply of money

M_d = Demand for money.

11. Money Supply and Banking System

i. High powered money (H) = Monetary liabilities of the Central bank + Government money

ii. Multiplier (M) = $\frac{1 + C_u}{C_u + r}$

Where,

C_u = Currency deposit ratio

r = Cash reserve ratio

iii. Money supply (M_s) = $H \times m$

Where,

H = High powered money

m = Money multiplier

iv. Finance Ratio = $\frac{\text{Total Issues}}{\text{National Income}}$

v. Financial Interrelation Ratio (FIR) = $\frac{\text{Total Issues}}{\text{Net Capital Formation}}$

vi. New Issue Ratio (NIR) = $\frac{\text{Primary Issues}}{\text{Net Capital Formation}}$

vii. Intermediation Ratio (IR) = $\frac{\text{Secondary Issues}}{\text{Primary Issues}}$

viii. Velocity of money (v) = $\frac{Y}{M_s}$

Where,

Y = Income

M_s = Money supply.

12. The Open Economy and Balance of Payments

i. Trade balance = Exports – Imports

ii. Current account balance

= Credit (Current account) – Debit (Current account)

iii. Capital account balance

= Credit (Capital account) – Debit (Capital account).

13. Modern Macro Economics: Fiscal Policy, Budget Deficits and Government Debt

i.	Fiscal Deficit	=	Borrowings and other liabilities
ii.	Primary Deficit	=	Fiscal deficit – Interest payments
iii.	Revenue Deficit	=	Revenue expenditure – Revenue receipts.

III. Financial Management

1. Time Value of Money

i. Future Value of a Lump Sum (Single Flow)

$FV_n = PV(1 + k)^n$

Where,

FV_n = Future value of the initial flow n years hence

PV = Initial cash flow

k = Annual rate of interest

n = Life of investment

ii. Effective rate of interest

$$r = \left(1+\frac{k}{m}\right)^m - 1$$

Where,

r = Effective rate of interest

k = Nominal rate of interest

m = Frequency of compounding per year

iii. Future Value Interest Factor of Annuity

$$FVIFA(k,n) = \frac{(1+k)^n - 1}{k}$$

Where,

k = Rate of interest

n = Time horizon

iv. Sinking Fund Factor = $\frac{1}{FVIFA(k,n)}$

Where,

FVIFA(k,n) = Future value interest factor for annuity at k% for n years

v. Present Value Interest Factor of Annuity

$$PVIFA(k,n) = \frac{(1+k)^n - 1}{k(1+k)^n}$$

Where,

k = Rate of interest

n = Time horizon

vi. Capital Recovery Factor = $\frac{1}{PVIFA(k,n)}$

Where,

k = Rate of interest

n = Time horizon

vii. Present Value Interest Factor of a Perpetuity

P_∞ = 1/k

Where,

k = Rate of interest.

2. Risk and Return

i. Rate of return

$$k = \frac{D_t + (P_t - P_{t-1})}{P_{t-1}}$$

Where,

k	=	Rate of return
P_t	=	Price of security at time 't', i.e., at the end of the holding period
P_{t-1}	=	Price of the security at time 't – 1' i.e., at the beginning of the holding period or purchase price
D_t	=	Income or cash flows receivable from the security at time 't'

ii. Expected rate of return $(\bar{k}) = \sum_{i=1}^{n} p_i k_i$

Where,

k_i	=	Rate of return from the ith outcome
p_i	=	Probability of the ith outcome
n	=	Number of possible outcomes
i	=	Outcome i

iii. Variance of an asset's rate of return, VAR(k) = $\sum_{i=1}^{n} p_i (k_i - \bar{k})^2$

Where,

VAR (k)	=	Variance of returns
p_i	=	Probability associated with ith possible outcome
k_i	=	Rate of return from the ith possible outcome
$\bar{k}$	=	Expected rate of return
n	=	Number of years
i	=	Outcome i

iv. Standard deviation, $\sigma = \sqrt{VAR(k)}$

v. CAPM model:

$k_j = r_f + \beta_j (k_m - r_f)$

Where,

k_j	=	Expected or required rate of return on security 'j'
r_f	=	Risk-free rate of return
β_j	=	Beta coefficient of security 'j'
k_m	=	Return on market portfolio

vi. Beta of security i, $\beta_i = \frac{Cov_{im}}{\sigma_m^2}$

Where,

Cov_{im}	=	Covariance of security i with true market
σ_m^2	=	Variance of returns on the market index

vii. Alpha of security i (α) $= E(r_i) - R(r_i)$

$= E(r_i) - \left[r_f + \beta_{im}\left(E\left(r_m\right) - r_f\right)\right]$

Where,

α	=	The difference between expected return and required return
r_f	=	Risk-free rate
β_{im}	=	Beta coefficient of security i
$E(r_i)$	=	Expected return of security i
$R(r_i)$	=	Required return from security i
$E(r_m)$	=	Return on market portfolio

viii. Systematic risk of security, i $= \beta_{im}^2 \sigma_m^2$

$$= \frac{\rho_{im}^2 \sigma_i^2 \sigma_m^2}{\sigma_m^2}$$

$= \rho_{im}^2 \sigma_i^2$

$= R_{im}^2 \sigma_i^2$

since $\left[R_{im}^2 = \rho_{im}^2\right]$

Where,

β_{im}	=	Beta coefficient of security i
σ_m^2	=	Market variance
σ_i^2	=	Variance of security i
ρ_{im}^2	=	The correlation coefficient, and
R_{im}^2	=	The coefficient of determination between the security i and the market portfolio

ix. Unsystematic Risk, (σ_{ei}^2) $= \sigma_i^2 - \beta_{im}^2 \sigma_m^2$

Or

$= \sigma_i^2 - \rho_{im}^2 \sigma_i^2$

$= \sigma_i^2 \left(1 - \rho_{im}^2\right)$

$= \sigma_i^2 \left(1 - R_{im}^2\right)$

Where,

σ_i^2	=	Variance of Security i
β_{im}	=	Beta coefficient of security i
σ_m^2	=	Market variance
ρ_{im}^2	=	The correlation coefficient, and
R_{im}^2	=	The coefficient of determination between the security i and the market portfolio.

3. Valuation of Securities

i. Equity Valuation:

a. The intrinsic value or present value of equity share

$$(P_0) = \sum_{t=1}^{n} \frac{D_t}{(1+k_e)^t} + \frac{P_n}{(1+k_e)^n}$$

Where,

P_0	=	Current market price of the equity share or intrinsic value of the share
D_t	=	Expected equity dividend at time t
P_n	=	Expected price of the equity share at time n
k_e	=	Expected rate of return or required rate of return
n	=	Investment period
t	=	Time t

b. The value of equity share when there is constant growth

$$P_0 = \frac{D_0(1+g)}{k_e - g}$$

Where,

D_0	=	Current dividend per share
g	=	Expected constant growth rate in dividends
k_e	=	Expected rate of return or required rate of return

ii. Bond Valuation:

a. The intrinsic value or the present value of a bond

V_0 or $P_0 = I(PVIFA_{kd,\ n}) + F(PVIF_{kd,\ n})$

Where,

V_0	=	Intrinsic value of the bond
P_0	=	Present value of the bond
I	=	Annual interest payable on the bond
F	=	Principal amount (par value) repayable at the maturity time
n	=	Maturity period of the bond
k_d	=	Cost of Capital or Required rate of return

b. Current yield = $\frac{\text{Coupon Interest}}{\text{Prevailing Market Price}}$

c. Yield to maturity **r** in the equation

$$P_0 = \sum_{t=1}^{n} \frac{I}{(1+r)^t} + \frac{F}{(1+r)^n}$$

Where,

n	=	Maturity period of the bond
I	=	Annual interest payable on the bond
F	=	Principal amount (par value) repayable at the maturity time

iii. Valuation of a Convertible:

$$\text{The value of convertible} = \sum_{t=1}^{n} \frac{C}{(1+r)^t} + \frac{(P_n) \times \text{Conversion ratio}}{(1+r)^n}$$

Where,

C = Coupon amount

r = Required rate of return

P_n = Expected price of equity share on conversion

n = Number of years to maturity.

4. Financial Statement Analysis

i. Liquidity Ratios:

a. Current Ratio = Current Assets/Current Liabilities

b. $$\text{Quick Ratio} = \frac{\text{Current Assets} - \text{Inventories}}{\text{Current Liabilities}}$$

c. $$\text{Bank finance to working capital ratio} = \frac{\text{Short-term bank borrowings}}{\text{Working capital gap}}$$

ii. Leverage Ratios:

a. $$\text{Long -term Debt-Equity Ratio} = \frac{\text{Long -term debt}}{\text{Net worth}}$$

b. $$\text{Total Debt-Equity Ratio} = \frac{\text{Total debt}}{\text{Net worth}}$$

c. $$\text{Debt-Asset Ratio} = \frac{\text{Total debt}}{\text{Total assets}}$$

iii. Coverage Ratios:

a. $$\text{Interest coverage ratio} = \frac{\text{EBIT}}{\text{Interest}}$$

Where,

EBIT = Earning before interest and tax

b. $$\text{Cash flow coverage ratio} = \frac{\text{EBILT} + \text{D}}{\text{I} + \text{L} + \frac{\text{LR}}{(1-t)} + \frac{\text{P}}{(1-t)}}$$

Where,

EBILT = Earnings before interest, lease payments and taxes

D = Depreciation

I = Interest charges

L = Lease payments

t = Marginal Tax Rate

LR = Loan Repayment

P = Preference dividend

c. Debt Service Coverage Ratio

$$= \left(\frac{\begin{array}{l}\text{PAT} + \text{Depreciation} + \text{Other non-cash charges} \\ + \text{Interest on term loan}\end{array}}{\text{Interest on term loan} + \dfrac{\text{Re payment of the term loan}}{1 - t}} \right)$$

Where,

PAT = Profit after tax

t = Marginal tax rate

iv. Turnover Ratios:

a. Inventory turnover $= \dfrac{\text{Cost of goods sold}}{\text{Average inventory}}$

b. Accounts receivables turnover

$$= \frac{\text{Net credit sales}}{\text{Average accounts receivable}}$$

c. Total assets turnover $= \dfrac{\text{Net sales}}{\text{Average total assets}}$

v. Profitability Ratios:

a. Gross profit margin $= \dfrac{\text{Gross profit}}{\text{Net sales}}$

b. Net profit margin $= \dfrac{\text{Profit after tax}}{\text{Net sales}}$

c. Return on investment (Earning Power) $= \dfrac{\text{EBIT}}{\text{Average total assets}}$

Where,

EBIT = Earning before interest and tax

d. Return on Net Worth $= \dfrac{\text{Profit after tax}}{\text{Average net worth}}$

5. Financial Forecasting

i. External financing requirement

$$\text{EFR} = \frac{A}{S}(\Delta S) - \frac{L}{S}(\Delta S) - mS_1(1 - d)$$

Where,

EFR = External financing requirement

A/S = Current assets and fixed assets as a proportion of sales

ΔS = Expected increase in sales

L/S = Spontaneous liabilities as a proportion of sales

m = Net profit margin

S_1 = Projected sales for next year

d = Dividend pay-out ratio

ii. Sustainable growth rate (g) $= \dfrac{m(1-d)A/E}{A/S_0 - m(1-d)A/E}$

Where,

m = Net profit margin

d = Dividend pay-out ratio

g = Sustainable growth rate with internal equity

A/E = $\dfrac{\text{Total Assets}}{\text{Equity}}$ = Current and fixed assets as proportion of equity

A/S_0 = Current and fixed assets as proportion of sales at time 0.

6. Leverages

i. Degree of Operating Leverage (DOL) = [Q(S – V)] / [Q(S – V) – F]

Where,

Q = Quantity sold

S = Selling price per unit

V = Variable cost per unit

F = Total fixed deposit

ii. Degree of Financial Leverage (DFL) $= \dfrac{EBIT}{EBIT - I - \dfrac{D_p}{(1-T)}}$

Where,

I = Interest amount

D_p = Preference dividend

T = Tax rate

EBIT = Earnings Before Interest and Tax

iii. Degree of Total Leverage (DTL) $= DOL \times DFL$

$$= \frac{Q(S-V)}{Q(S-V) - F - I - \dfrac{D_p}{(1-T)}}$$

Where,

DOL = Degree of operating leverage

DFL = Degree of financial leverage

Q = Quantity sold

S = Selling price per unit

V = Variable cost per unit

F = Total fixed deposit

I = Interest amount

D_p = Preference dividend

T = Tax rate

iv. Overall break-even point (Q) $= \dfrac{F + I + \dfrac{D_p}{(1-T)}}{(S-V)}$

Where,

S	=	Selling price per unit
V	=	Variable cost per unit
F	=	Total fixed deposit
I	=	Interest amount
D_p	=	Preference dividend
T	=	Tax rate

v. Operating break-even point (Q) $= \dfrac{F}{(S-V)}$

Where,

S	=	Selling price per unit
V	=	Variable cost per unit
F	=	Total fixed deposit

vi. Financial break-even point (EBIT) = $I + \dfrac{D_P}{(1-T)}$

Where,

I	=	Interest amount
D_p	=	Preference dividend
T	=	Tax rate.

7. Cost of Capital

i. Cost of Term Loans = I (1 – T)

Where,

I	=	Interest rate
T	=	Tax rate

ii. Cost of Debentures, $P = \sum_{t=1}^{n} \dfrac{I(1-t)}{(1+k_d)^t} + \dfrac{F}{(1+k_d)^n}$

Where,

k_d	=	Post-tax cost of debenture capital
I	=	Annual interest payment per debenture capital
t	=	Corporate tax rate
F	=	Redemption price per debenture
P	=	Net amount realized per debenture
n	=	Maturity period

iii. Cost of Preference Capital, $P = \sum_{t=1}^{n} \dfrac{D}{(1+k_p)^t} + \dfrac{F}{(1+k_p)^n}$

Where,

k_p	=	Cost of preference capital
D	=	Preference dividend per share payable annually
F	=	Redemption price

P = Net amount realized per share

n = Maturity period

iv. Cost of Equity Capital

a. Dividend forecast approach, $P_e = \frac{D_1}{k_e - g}$

Where,

P_e = Price per equity share

D_1 = Expected dividend per share at the end of one year

k_e = Rate of return required by the equity shareholders

g = Growth rate of dividends

b. Cost of External Equity, $k'_e = \frac{D_1}{P_o(1-f)} + g$ (Method 1)

$k'_e = \frac{k_e}{(1-f)}$ (Method 2)

Where,

k'_e = Cost of external equity

k_e = Cost of equity

D_1 = Dividend expected at the end of year 1

P_o = Current market price per share

g = Constant growth rate applicable to dividends

f = Floatation costs as a percentage of the current market price

v. Weighted Average Cost of Capital

$$= k_e\left(\frac{E}{E+P+D}\right) + k_p\left(\frac{P}{E+P+D}\right) + k_d(1-T)\left(\frac{D}{E+P+D}\right)$$

Where,

E = Market value of equity

P = Market value of preference capital

D = Market value of debt

k_e = Cost of equity

k_p = Cost of preference capital

k_d = Cost of debt

T = Tax rate.

8. Capital Structure

i. Overall capitalization rate of the firm

$$k_o = k_d \frac{B}{B+S} + k_e \frac{S}{B+S}$$

Where,

k_d = The cost of debt

B = The market value of the outstanding debt

S = The market value of equity

k_e = The cost of equity

k_o = The weighted average cost of capital

ii. Present value of a tax shield of interest payments:

a. When debt is perpetual = t_cB

Where,

t_c = The tax rate on corporate income

B = The market value of the debt

b. When corporate taxes are considered the value of the levered firm

$$V = \frac{O(1-t_c)}{k} + t_c B$$

Where,

O = Operating income

t_c = The tax rate on corporate income

B = The market value of the debt

k = Interest rate on debt

c. If the personal tax rate is t_p, the tax advantage of debt = $t_c B (1 - t_p)$

Where,

t_c = The tax rate on corporate income

B = The market value of the debt

d. When the tax rate on stock income (t_{ps}) differs from the tax rate on debt income (t_{pd}),

the tax advantage of debt capital $= 1 - \frac{(1-t_c)(1-t_{ps})}{(1-t_{pd})} \times B$

Where,

t_c = The tax rate on corporate income

B = The market value of the debt.

9. Dividend Policy

i. Traditional Model (Graham-Dodd Model), P = m (D + E/3)

Where,

P = The market price per share

m = The multiplier

D = The dividend per share

E = The earnings per share

ii. Walter Model, $P = \frac{D + (E-D)r/k_e}{k_e}$

Where,

P = The market price per share

D = The dividend per share

E = The earnings per share

r = The internal rate of return

k_e = The cost of equity capital

iii. Gordon Model, $P_0 = \frac{Y_0(1-b)}{k_e - br}$

Where,

P_0 = The market price per share at the beginning of period 0

Y_0 = The earnings per share for period 0

b = The retention ratio (retained earnings/total earnings)

r = The return on investments

k_e = The cost of equity capital or (Cost of capital of firm)

iv. MM Approach, $P_0 = \dfrac{D_1 + P_1}{1 + k_e}$

Where,

P_0 = The market price per share at the beginning of period 0

D_1 = The expected dividend per share for period 1

P_1 = The market price per share at the end of period 1

k_e = The cost of equity capital

v. Corporate Dividend Behavior (Lintner Model)

$D_t = cr\ EPS_t + (1 - c)\ D_{t-1}$

Where,

D_t = The dividend per share for the time period t

c = The weightage given to current earnings by the firm

r = The target pay-out rate

EPS_t = The earnings per share for the time period t

D_{t-1} = The dividend per share for the time period (t – 1).

10. Estimation of Working Capital Needs

i. Durations at various stages of production

a. Raw Material Storage Period = $\dfrac{\text{Average Stock of Raw Material and Stores}}{\text{Average Raw Materials and Stores consumed per day}}$

b. Work-in-process period = $\dfrac{\text{Average Work-in-process inventory}}{\text{Average daily cost of production}}$

c. Finished goods storage period = $\dfrac{\text{Average finished good inventory}}{\text{Average daily cost of sales}}$

d. Average collection period = $\dfrac{\text{Average accounts receivable}}{\text{Average daily credit sales}}$

e. Average payment period = $\dfrac{\text{Average accounts payable}}{\text{Average credit purchases per day}}$

ii. Net operating cycle period = a + b + c + d – e

iii. Weighted Operating Cycle =

$D_{woc} = W_{rm} D_{rm} + W_{wip} D_{wip} + W_{fg} D_{fg} + W_{ar} D_{ar} - W_{ap} D_{ap}$

D_{woc} = Duration of weighted operating cycle

W_{rm} = Weight of raw material expressed as a percentage of raw material cost to sales

D_{rm} = Duration of raw material

W_{wip}	=	Weight of work-in-progress expressed as a percentage of work-in-progress cost to sales
D_{wip}	=	Duration of work-in-progress
W_{fg}	=	Weight of finished goods expressed as a percentage of cost of goods sold to sales
D_{fg}	=	Duration of finished goods
W_{ar}	=	Weight of accounts receivables expressed as a percentage of sales to sales
D_{ar}	=	Duration of accounts receivables
W_{ap}	=	Weight of accounts payables expressed as a percentage of raw material cost to sales
D_{ap}	=	Duration of accounts payables.

11. Inventory Management

i. Economic Order Quantity

$$EOQ = \sqrt{\frac{2UF}{PC}} \text{ units}$$

Where,

U	=	Annual usage rate
F	=	Ordering cost
C	=	Carrying cost
P	=	Price per unit

ii. Reorder point = $S \times L + F\sqrt{(S \times R \times L)}$

Where,

S	=	Usage in units
L	=	Lead time in days
R	=	Average number of units per order
F	=	Stock out acceptance factor.

12. Receivables Management

i. Effect of relaxing the credit standards on profit

$$\Delta P = \Delta S(1 - V) - k\,\Delta I - b_n\,\Delta S$$

Where,

ΔP	=	Change in profit
ΔS	=	Increase in sales
V	=	Variable costs to sales ratio
k	=	Cost of capital
ΔI	=	Increase in investment in receivables
	=	$\frac{\Delta S}{360} \times \text{Average collection period} \times V$
b_n	=	Bad debts loss ratio on new sales
$1 - V$	=	Contribution to sales ratio

ii. Effect of increasing the credit period on profit

$\Delta P = \Delta S\,(1 - V) - k\,\Delta I - b_n\,\Delta S$

The components of the formula are same excepting

$$\Delta I = (ACP_N - ACP_O)\left[\frac{S_0}{360}\right] + V(ACP_N)\frac{\Delta S}{360}$$

Where,

ΔI	=	Increase in investment in receivables
ACP_N	=	New ACP (after increasing credit period)
ACP_O	=	Old ACP
V	=	Ratio of variable cost to sales
ΔS	=	Increase in sales
k	=	Cost of capital
S_0	=	Sales before increasing the credit period

iii. The effect on profit for a change in cash discount rate

$\Delta P = \Delta S\,(1 - V) + k\Delta I - \Delta\,DIS$

Where,

ΔS	=	Increase in sales
V	=	Ratio of variable cost to sales
k	=	Cost of capital
ΔI	=	Savings in investment in receivables
	=	$\frac{S_0}{360}(ACP_O - ACP_N) - V\,\frac{\Delta S}{360}\,ACP_N$
ΔDIS	=	Increase in discount cost
	=	$p_n\,(S_0 + \Delta S)d_n - p_o\,S_0\,d_o$

Where,

p_n	=	Proportion of discount sales after liberalizing
S_o	=	Sales before liberalizing
ΔS	=	Increase in sales
d_n	=	New discount percentage
p_0	=	Proportion of discount sales before liberalizing
d_0	=	Old discount percentage
ACP_O	=	Average collection period before increasing cash discount
ACP_N	=	Average collection period after increasing cash discount

iv. Effect of decreasing the rigor of collection program on profit:

$\Delta P = \Delta S(1 - V) - k\Delta I - \Delta BD$

Where,

ΔP	=	Change in profits
ΔS	=	Increase in sales
V	=	Variable costs to sales ratio
k	=	Cost of capital

ΔI = Increase in investment in receivables

$$= \frac{S_o}{360}(ACP_N - ACP_O) + \frac{\Delta S}{360} ACP_N \times V$$

ΔBD = Increase in bad debts cost

$$= b_n (S_o + \Delta S) - b_o S_o$$

ACP_O = Average collection period before relaxing collection effort

ACP_N = Average collection period after relaxing collection effort

b_o = Proportion of bad debts to sales before relaxing collection effort

b_n = Proportion of bad debts to sales after relaxing collection effort.

13. Cash Management

i. Baumol Model, TC = I (C/2) + b (T/C)

Where,

TC = Total costs (total conversion costs + total holding costs)

I = Interest rate on marketable securities per planning period

C = Amount of securities liquidated per batch

T = Estimated cash requirement over the planning period

b = Fixed conversion cost

The point where total costs are minimum:

$$C = \sqrt{\frac{2bT}{I}}$$

ii. Miller and Orr Model, $RP = \sqrt[3]{\frac{3b\sigma^2}{4I}} + LL$ and,

$$UL = 3\,RP - 2\,LL$$

Where,

LL = Lower control limit

RP = Return point

UL = Upper control limit

b = Fixed conversion cost

I = Interest rate per day on marketable securities.

14. Capital Expenditure Decisions

i. Accounting Rate of Return

$$(ARR) = \frac{\text{Average profit after tax}}{\text{Average book value of the investment}}$$

ii. Net Present Value (NPV)

$$NPV = \sum_{t=1}^{n} \frac{CF_t}{(1+k)^t} - I_0$$

Where,

k = Cost of funds

CF_t = Cash flows at the end of the period t

I_0 = Initial investment

n = Life of the investment

iii. Benefit-Cost Ratio (BCR)

$$BCR = \frac{PV}{I}$$

Where,

BCR = Benefit-Cost Ratio

PV = Present Value of future cash flows

I = Initial investment

iv. Net-benefit-cost Ratio

$$NBCR = \frac{NPV}{I}$$

Where,

NPV = Net present value

I = Initial investment

v. Internal Rate of Return (IRR)

$$I_0 = \sum_{t=1}^{n} \frac{CF_t}{(1+k)^t}$$

Where,

k = IRR, which is that rate of return where $\sum_{t=1}^{n} \frac{CF_t}{(1+k)^t} - I_0 = 0$

CF = Cash flow

I_0 = Initial investment

n = Life of investment.

IV. Financial Risk Management

1. Corporate Risk Management

i. Historical (ex-post)

a. Arithmetic mean return, $\bar{r}_i = \frac{1}{n}\sum_{t=1}^{n} r_{it}$

b. Variance (risk), $\sigma_i^2 = \frac{1}{n-1}\sum_{t=1}^{n}(r_{it} - \bar{r}_i)^2$

c. Standard deviation, $\sigma_i = \sqrt{\text{Variance}}$

ii. Expected (ex-ante)

a. Expected return, $E(r_i) = \sum_{s=1}^{n} r_{is} P_s$

b. Variance (Risk), $\sigma_i^2 = \sum_{s=1}^{n}[r_{is} - E(r_i)]^2 . P_s$

Where,

$\bar{r}_{it}$ = Historical (ex post) return generated by the ith stock in time period t

r_{is} = Expected (ex ante) return for the ith stock assuming that S state of the world occurs

P_s = Probability that the S state of the world will occur

r_i = Return on a security 'i'

iii. Estimated return on a stock $(R_s) = \alpha + \beta r_m$

Where,

r_m = Return on market

β = Measure of stock's sensitivity to the market index

α = Estimated return when the market return is zero

iv. According to the CAPM, the required return on a security

$R_s = R_f + \beta(R_m - R_f)$

Where,

R_f = Return on risk-free investment

R_m = Return on market

β = Measure of stock's sensitivity to the market index.

2. Futures

i. Effective price = $Sp_2 + (Ft_1 - Ft_2)$

If bases remains the same

Effective price = Sp_1

Where,

Sp_1 = Spot price at time t_1

Sp_2 = Spot price at time t_2

Ft_1 = Futures price at time t_1

Ft_2 = Futures price at time t_2

Basis = Current cash price – Futures price

ii. Margin

Initial margin = $\mu + 3\sigma$

Where,

μ = Mean

σ = Standard Deviation

iii. Relationship between the cash price and the futures price of any commodity:

$$F_{t,T} = C_t + C_t \times S_{t,T} \times \frac{T-t}{365} + G_{t,T}$$

Where,

C_t = Cash price at time t

$S_{t,T}$ = Annualized interest rate on borrowings

$G_{t,T}$ = Storage costs

$T-t$ = Time period

$F_{t,T}$ = The futures price at time t, which is to be delivered at time period T

iv. $\text{Hedge Ratio (HR)} = \frac{\text{Futures position}}{\text{Underlying asset position}}$

v. Minimum variance hedge ratio, $h = F_p \frac{\sigma_{Sp}}{\sigma_{Ft}}$

Where,

h = Hedge ratio

F_p = Coefficient of correlation between S_p and F_t

σ_{Ft} = Standard deviation of ΔF_t

σ_{Sp} = Standard deviation of ΔS_p

ΔF_t = Change of futures price during hedging

ΔS_p = Change in spot price during hedging

vi. $\text{T-bill purchase price} = \text{Face value} \times \left[1 - \frac{\%\text{ discount}}{100} \times \frac{\text{Days to maturity}}{360}\right]$

vii. IRR (Implied Repo Rate)

IRR = $(FP_{t,T} - CP_{t,T})/(CP_{t,T}) \times 360/T - t$

Where,

$FP_{t,T}$ = Price of futures T-bill

$CP_{t,T}$ = Cash price of T-bill

$T - t$ = Time period

viii. Transaction price or cash price of the bond,

P = Quoted price + Accrued interest

Invoice price = (Futures settlement price × Conversion factor) + Accrued interest

ix. $HR = -\left(\frac{\text{Cash market principal}}{\text{Futures market principal}}\right) \times \text{Conversion factor}$

$$HR = \left(\frac{\text{Cash flow to be hedged}}{\text{Value of futures contract}} \times \text{Conversion factor} \times \frac{\text{Portfolio duration}}{\text{CTD bond duration}} \right)$$

x. Change in value of a bond,

$$dB = -\frac{\text{Duration}}{1+y} \times B \times dy$$

Where,

B = Value of the bond

y = Yield to maturity

dy = Change in yield

xi. Basis point value,

$$BPV = \frac{\text{Duration}}{(1+y/2)} \times \text{Market value of bond} \times 1\text{bp}$$

$$HR = \frac{BPV\,(\text{target}) - BPV\,(\text{existing})}{BPV\,(\text{futures})}$$

xii. $$N_f = -\left(\frac{DUR_s - DUR_T}{DUR_f}\right)\left(\frac{S}{f}\right)\left(\frac{1+y_f}{1+y_s}\right)$$

Where,

N_f = Number of futures contract required to change the duration to DUR_T

DUR_s = Duration of bond with face value S

DUR_f = Duration of futures contract with price f

DUR_T = Target portfolio duration

y_f = Yield implied by futures price

y_s = Yield implied by spot portfolio

xiii. Treasury bond implied repo rate $= \left[\frac{f_T\,(CF_T) + AI_T}{f_t\,(CF_t) + AI_t}\right]^{1/(T-t)} - 1$

Where,

CF_t = Conversion factor for bond delivered at t

CF_T = Conversion factor for bond delivered at T

f_t = Today's futures price for contract expiring at t

f_T = Today's futures price for contract expiring at T

AI_t = Accrued interest on bond as of time t

AI_T = Accrued interest on bond as of time T

T – t = Time period.

3. Options

i. Pay-off from Buying a call option = Max (S – E, 0)

Pay-off from Buying a put option = Max (E – S, 0)

Where,

S = The market price of the underlying asset

E = The exercise price

ii. Margin

a. Margin is higher of the following for naked out of the money option

Margin = Contract size × Option premium per share + 0.2 (Market value of share) × Contract size – Contract size (amount by which contract is out-of-the money)

Margin = Contract size × Option premium per share + 0.10 (stock's price) x Contract size

b. Margin for naked option (in-the-money)

= Contract size × Option premium per share + 0.20 (stock's market price) × Contract size

iii. Option price is a function of

C_o or P_o = $f(S_o, E, \sigma^2, t, r_f, d)$

Where,

C_o = Value of call option

P_o = Value of put option

f = Function of

E = Exercise price

S_o = Price of underlying stock

σ^2 = Price volatility of underlying stock

t = Time to expiration

r_f = Risk-free interest rate

d = Cash dividend

iv. Put-call parity equation

$C + Xe^{-r(T-t)} = P + S$

Where,

C = Call price

$Xe^{-r(T-t)}$ = Present value of exercise price

P = Put price

S = Current market price

v. Binomial Pricing

Call price, $C = \dfrac{C_u p + C_d(1-p)}{R}$

$p = \dfrac{R-d}{u-d}$

Where,

u = 1 + percentage increase in stock price from time 0 to time t

d = 1+ percentage decrease in stock price from time 0 to time t

C = The call price

C_u = The value of the call if the stock price increases

C_d = The value of call if the stock price decreases

R = 1+ risk-free rate of return (r)

p = Probability of price increase

vi. Black-Scholes option pricing model:

a. For a non-dividend paying stock

$C = S_0N(d_1) - Xe^{-r(T-t)}N(d_2)$

$P = Xe^{-r(T-t)}N(-d_2) - S_0N(-d_1)$

Where,

$$d_1 = \frac{\ln(S_0/X) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{(T-t)}}$$

$$d_2 = \frac{\ln(S_0/X) + \left(r - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{(T-t)}}$$

Or,

d_2	=	$d_1 - \sigma\sqrt{T-t}$
C	=	The call option price
P	=	The put option price
S_0	=	The spot price of the underlying asset
$Xe^{-r(T-t)}$	=	Present value of exercise price
r	=	The risk-free rate
(T – t)	=	The time to expiration expressed in years
σ	=	The annualized standard deviation of returns on the underlying asset, i.e., the volatility measure
N(d)	=	Cumulative standard normal distribution
e	=	Exponential function
In	=	Natural logarithm

b. For a dividend paying stock:

$C = S_0e^{-qt}N(d_1) - Xe^{-rt}N(d_2)$

$P = Xe^{-rt}N(-d_2) - S_0e^{-qt}N(-d_1)$

Where,

$$d_1 = \frac{\ln(S_0/X) + \left(r - q + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

Or,

d_2	=	$d_1 - \sigma\sqrt{t}$
q	=	Dividend yield
C	=	The call option price
P	=	The put option price
S_0	=	The spot price of the underlying asset
$Xe^{-r(T-t)}$	=	Present value of exercise price
r	=	The risk-free rate
σ	=	The annualized standard deviation of returns on the underlying asset, i.e., the volatility measure
N(d)	=	Cumulative standard normal distribution
e	=	Exponential function
In	=	Natural logarithm

c. For a currency option:

$C = S_0 e^{-r_f t} N(d_1) - Xe^{-rt} N(d_2)$

$P = Xe^{-rt} N(-d_2) - S_0 e^{-r_f t} N(-d_1)$

Where,

$$d_1 = \frac{\ln(S_0/X) + \left(r - r_f + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

C = The call option price

P = The put option price

r = Domestic risk free rate

r_f = Foreign risk free rate

S_0 = The spot price of the underlying asset

X = The strike price of the option

$N(d)$ = Cumulative standard normal distribution

e = Exponential function

In = Natural logarithm.

4. Swaps

i. Valuation of interest rate swaps, $V = F_B - F_F$

V = Value of the swap

F_B = Value of fixed coupon bond

F_F = Value of floating rate bond

ii. Valuation of currency swaps, $V = P_F - P_L$

V = Value of the swap

P_F = Value of foreign currency bond

P_L = Value of local currency bond.

5. Sensitivity of Option Premiums

i. Delta call = $\Delta C/\Delta S = N(d_1)$

Where,

ΔC = Change in the call price

ΔS = Change in the stock price

ii. Delta put = $\Delta C/\Delta S = N(d_1) - 1$

iii. Delta for portfolio of derivatives consisting of a single underlying asset:

$$\Delta_P = \sum_{j=1}^{n} W_j \Delta_j$$

Where,

Δ_P = Δ of portfolio

Δ_j = Δ of j derivative

W_j = Weight of j derivative in the portfolio

iv. Theta of call $= \frac{-SN'(d_1)\sigma}{2\sqrt{T-t}} - rXe^{-r(T-t)} N(d_2)$

S = The spot price of the underlying asset

$N'(d_1) = \frac{1}{\sqrt{2\pi}} e^{-d_1^2/2}$

σ = The annualized standard deviation of returns on the underlying asset, i.e., the volatility measure

(T – t) = The time to expiration expressed in years

v. Theta of put = $\frac{-SN'(d_1)\sigma}{2\sqrt{T-t}} + r\,Xe^{-r(T-t)}\,N(-d_2)$

Where,

d_1 and d_2 are defined as per Black-Scholes model.

$N'(d) = \frac{1}{\sqrt{2\pi}} e^{-d^2/2}$

σ = The annualized standard deviation of returns on the underlying asset, i.e., the volatility measure

S = The spot price of the underlying asset

(T – t) = The time to expiration expressed in years

vi. Vega of call or put = $S\sqrt{T-t}\;\; N'(d_1)$

Where,

$N'(d_1) = \frac{1}{\sqrt{2\pi}} e^{-d_1^2/2}$

S = The spot price of the underlying asset

(T – t) = The time to expiration expressed in years

vii. Rho for a European put option = $-X(T-t)e^{-r(T-t)}N(-d_2)$

viii. Rho for a European call option = $X(T-t)e^{-r(T-t)}N(d_2)$

ix. Gamma of call or put = $N'(d_1)/S\sigma\sqrt{T-t}$

x. Portfolio Insurance:

Delta of a put on an index

$\Delta = e^{-q(T-t)}\,[N(d_1) - 1]$

$d_1 = \frac{\ln(S/X) + (r - q + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$

r = Domestic Risk-Free rate

q = Dividend yield.

6. Value at Risk

i. Daily volatility = $\frac{\text{Annual volatility}}{\sqrt{\text{Number of working days}}}$

Daily value-at-Risk (VaR) (at a confidence level x%)

= Position Value x Daily Volatility × K(x)

Where,

K(x) = Factor relating to x% of confidence level.

V. International Finance

1. The Foreign Exchange Market

i. The conditions for no arbitrage possibility

a. $(A/B)_{ask} \times (B/C)_{ask} \times (C/A)_{ask} \geq 1$

b. $(A/B)_{bid} \times (B/C)_{bid} \times (C/A)_{bid} \leq 1$

ii. The annualized percentage premium on currency B for quote (A/B)

$$\frac{\text{Forward}(A/B)_{mid} - \text{Spot}(A/B)_{mid}}{\text{Spot}(A/B)_{mid}} \times \frac{12}{m} \times 100$$

Where,

m = Maturity of the forward contract in months.

2. Exchange Rate Determination

i. Interest rate parity (Investor's decision)

a. Investment in currency A is profitable, if

$$(1 + r_A) > \frac{F(A/B)}{S(A/B)} \times (1 + r_B)$$

b. Investment in currency B is profitable, if

$$(1 + r_A) < \frac{F(A/B)}{S(A/B)} \times (1 + r_B)$$

c. The investor would be indifferent to the choice of currencies, if $(1 + r_A) = \frac{F(A/B)}{S(A/B)} \times (1 + r_B)$

Where,

F(A/B) = Forward rate of currency B expressed in terms of currency A

S(A/B) = Spot rate of currency B expressed in terms of currency A

r_A, r_B = Investment rates in currencies A and B respectively

ii. Interest rate parity (Borrower's decision)

a. Borrowing in currency A is profitable, if $(1 + r_A) < \frac{F(A/B)}{S(A/B)} \times (1 + r_B)$

b. Borrowing in currency B is profitable, if $(1 + r_A) > \frac{F(A/B)}{S(A/B)} \times (1 + r_B)$

c. The borrower would be indifferent to the choice of currency, if $(1 + r_A) = \frac{F(A/B)}{S(A/B)} \times (1 + r_B)$

Where,

F(A/B) = Forward rate of currency B expressed in terms of currency A

S(A/B) = Spot rate of currency B expressed in terms of currency A

r_A, r_B = Borrowing interest rates in currencies A and B respectively.

3. International Project Appraisal

i. The adjusted present value of a foreign project

$$APV = -S_0\,(C_0 - A_0) + \sum_{t=1}^{n} \frac{(S_t^* C_t^* + E_t^*)(1-T)}{(1+k_e)^t}$$

$$+ \sum_{t=1}^{n} \frac{D_t\,T}{(1+k_d)t} + \sum_{t=1}^{n} \frac{rB_0\,T}{(1+k_b)^t} + S_0\left[CL_0 - \sum_{t=1}^{n} \frac{R_t}{(1+k_c)^t}\right]$$

$$+ \sum_{t=1}^{n} \frac{P_t^*\,T}{(1+k_p)^t} + \sum_{t=1}^{n} \frac{I_t}{(1+k_i)^t}$$

Where,

APV	=	Adjusted Present Value
S_0	=	Current exchange rate
C_0	=	Initial cash outlay in foreign currency terms
A_0	=	Activated funds
S_t^*	=	Expected exchange rate at time ‘t’
n	=	Life of the project
C_t^*	=	Expected cash flow at time ‘t’, in foreign currency terms
E_t^*	=	Expected effect on the cash flows of other divisions at time ‘t’, expressed in domestic currency terms; can be either positive or negative
T	=	Domestic or foreign tax rate, whichever is higher
D_t	=	Depreciation in home currency terms at time ‘t’. (If the depreciation is not allowed to be set off by the parent company against its own profits, it needs to be defined in foreign currency terms with its present value being converted at S_0 into domestic currency terms)
B_0	=	Contribution of the project to borrowing capacity of the parent firm.
r	=	Domestic interest rate
CL_0	=	Amount of concessional loan received in foreign currency
R_t	=	Repayment of concessional loan at time ‘t’
P_t^*	=	Expected savings at time ‘t’ from inter-subsidiary transfer pricing
I_t	=	Illegally repatriated cash flows at time ‘t’
k_e	=	All-equity discount rate, reflecting all systematic risks, including country risk and exchange-rate risk
k_d	=	Discount rate for depreciation allowances
k_b	=	Discount rate for tax savings from generation of borrowing capacity
k_c	=	Discount rate for tax savings due to concessionary loans, generally the interest rate in the absence of concessionary loans
k_p	=	Discount rate for savings through transfer pricing
k_i	=	Discount rate for illegal transfers.

4. International Equity Investments

i. Variance of domestic currency returns on foreign investment

$$= Var(r_f) + Var(\tilde{S}) + 2\,Cov(r_f, \tilde{S})$$

ii. According to international CAPM, the return on a security

$r_i = r_f + \beta_w (r_w - r_f)$

Where,

r_f = World risk-free rate of return

βw = World beta of the security

$= \frac{Cov(r_i, r_w)}{Var(r_w)}$

r_w = Return on the world-market portfolio.

5. Short-term Financial Management

i. The break-even-size of investment

$E = M[(k - i)/(k - d)]$

Where,

E = Surplus funds at break-even level

M = Minimum lot of investment

k = Interest rate on borrowed funds

i = Rate of interest for investment

d = Rate of interest for deposit.

VI. Investment Banking and Financial Services

1. Money Market

i. Annual Turnover of Primary Dealer/Satellite Dealer

$$= \frac{\text{Total Purchases and Sales during the year}}{\text{Average month-end stocks during the year}}$$

2. Rights Issues

i. Value of a Share after the Rights Issue = $\frac{NP_0 + S}{N+1}$

Where,

N = Number of existing shares for a rights share

P_0 = Cum-rights market price per share

S = Subscription price at which the rights shares are issued

ii. Value of a Right (R) = $\frac{P_r - S}{N+1}$

Where,

R = Value of a right

P_r = Market value of share trading with rights

S = Strike price

N = Number of rights to purchase a new share

iii. Share Price Ex-Rights

a. Market Value of each Right after the Rights Issue, R = $\frac{P_e - S}{N}$

b. Value of Shareholding after Subscription = $NP_0 + S$

Where,

P_e = Price of share ex-rights

S = Strike price

N = Number of rights to purchase a new share

P_0 = Cum-rights market price per share.

3. Lease Evaluation

i. Lessee's Angle

a. Weingartner's Model:

$\Delta NPV(L)$ = Initial Investment – P.V. (Lease Rentals) – Management Fee + P.V. (Tax Shield on Lease Rentals) + P.V. (Tax Shield on Management Fee) – P.V. (Tax Shield on Depreciation) – P.V. (Net Salvage Value).

b. Equivalent Loan Model:

Net Value of Lease = + Initial Investment – P.V. (Lease Payment Discounted at K_d) + P.V. (Tax Shield on Lease Payments Discounted at k_d) – P. V. (Depreciation Tax Shield discounted at k_d) – P.V. (Net Salvage value Discounted at k_d) – P.V. (Interest Tax Shield on displaced Debt Discounted at k_d)

Where,

K_d = Pre-tax marginal cost of debt

k_d = Post-tax marginal cost of debt

= K_d (1 – T)

T = Marginal tax rate

c. Bower-Herringer-Williamson (BHW) Model:

Financial Advantage of Leasing [FA(L)] =

Initial Investment – P.V. of Lease Payments

Or

FA (L) = P.V. of Loan Payments – P.V. of Lease Payments

Operating Advantage of Leasing [OA (L)] =

P.V. of lease related tax shield – P.V. of loan related tax shield – P.V. of Residual Value

d. Bower's Model:

Cost of Purchase (COP) = Initial investment – P.V. (Tax Shields on depreciation discounted at an unspecified rate) – P.V. (Net salvage discounted at marginal cost of capital)

Cost of Leasing (COL) = P.V. (Lease Rentals discounted at pre-tax cost of debt) – P.V. (Tax Shield on lease rentals discounted at an unspecified rate) + P.V. (Tax Shield on interest discounted at an unspecified rate)

e. Suggested Framework for Lease Evaluation:

NAL = Investment Cost – P.V. (Lease Payments discounted at K_d) + P.V. (Tax Shields on Lease Payments Discounted at k)– Management Fee + P.V. (Tax Shield on Management Fee discounted at k) – P.V. (Depreciation Tax Shields discounted at k) – P.V.(Interest Tax Shields Discounted at k) – P.V.(Residual value discounted at k)

ii. Lessor's Angle

a. Present value of rental stream:

$$PV = L \times \left(\frac{(1+j)}{(1+i)}\right) + PVIFA_{(j,n)}$$

Where,

PV = Present value of rental stream

Where, rentals increase/decrease at constant rate p.a.

L = Lease rental per period

n = Duration of lease in years

j = [(i – g)/(1 + g)]

i = Pre-tax yield p.a.

g = Constant rate of increase/decrease p.a.

b. Net Advantage of Leasing (NAL):

NAL = – Initial Investment + P.V. (Lease Payments) – P.V. (Tax Lease Payments) + P.V. (Management Fee) – P.V. (Tax on Management Fee) + PV. (Tax shields on Depreciation) + P.V. (Net Salvage Value) – P.V. (Initial Direct Costs) + P.V. (Tax Shield on Initial Direct Costs)

c. Gross Yield:

The gross yield of a lease can be defined as that compounded rate of return (discount rate) that equates: P.V (Lease Rentals) + P.V (Residual Value) to Investment cost

Where, management fee and initial direct costs are involved the gross yield will be the discount rate that equates:

P.V. (Lease Rentals) + P.V. (Residual Value) + Management Fees = Investment Cost + Initial Direct Costs)

d. Add on yield = $\frac{\text{Annual charge for credit}}{\text{Initial investment}} \times 100$

e. IRR based pricing:

$i = i_F + i_e + i_d$

Where,

i = Risk adjusted rate of return

i_F = Risk-free rate of return

i_e = Premium for the risk characterizing the existing lease investments

i_d = Premium for the differential risk characterizing the lease investment under review

f. Value of the asset or the implied interest return earned by the lessor

$$= \sum_{t=1}^{mn} \frac{\text{Lease Payments}}{(1+R/n)^t} + \frac{\text{Lease Value}}{(1+R/n)^{mn}}$$

Where,

n = Length of the lease term

m = Number of lease payments in a year

R = Implied Interest Return

If the lease payments are made in advance, $\sum_{t=1}^{mn}$ would be changed to $\sum_{t=0}^{mn-1}$

g. Internal Rate of Return or After tax cost of leasing

$$= -A + \sum_{t=1}^{n} \frac{L_t}{(1+r)^t} + \frac{T(L_t - D_t)}{(1+r)^t} - \frac{RV}{(1+r)^n}$$

Where,

A = The cost of the asset to be leased

L_t = The periodic lease payments at the end of the each period

T = The corporate tax rate

n = The lease term

D_t = The depreciation that can be claimed for tax purpose

RV = The residual value of the asset.

4. Hire Purchase

i. From Hirer's Angle:

a. COHP = Down payment + P.V (Hire Payments) + Service Fee – P.V (Tax shields on charge for credit of Hire payments & Service Fee) – P.V (Tax shields on Depreciation) – P.V (Net salvage value)

b. COL = P.V (Lease payments) + Lease management fee – P.V (Tax shields on lease payments & lease management Fee)

ii. From Finance Company's Angle:

a. NPV (Lease Plan) = – Initial Investment – Initial Direct costs + P.V (Lease Rentals) + Lease Management Fee + P.V (Tax shields on Initial direct costs & Depreciation) + P.V (Net Salvage Value) – P.V (Tax liability on Lease Rentals and Lease Management Fee)

b. NPV (HP Plan) = – Loan amount – Initial Direct costs + Documentation & Service Fee + P. V (H.P installments) – P.V (Interest Tax on Finance Income) – P.V (Income Tax on Finance Income netted for interest tax) + P.V (Tax shield on initial Direct costs) – P.V (Income Tax on Documentation & Service Fee)

iii. Effective rate of interest:

If payments are made in arrears,

$$I_{(app)} = \frac{n}{n+1} \times 2F$$

If the payments are made in advance,

$$i_{app} = \frac{n}{n-1} \times 2F$$

Where,

F = Flat rate of interest per unit time

N = Total number of repayments

iv. Interest Rebate:

Rule of 78 method

$$R = \frac{t(t+1)}{n(n+1)} \times D$$

Where,

t = Number of level installment that are not due and outstanding

n = Total number of level installment

D = Total change for credit

R = Interest rebate

Under modified Rule of 78,

$$\text{Interest Rebate} = \frac{(t-\alpha)(t-\alpha+1)}{n(n+1)} \times D$$

Where,

α = Deferent period

Under Hire Purchase Act, 1972,

$$\text{Interest rebate} = \frac{2}{3} \times \frac{t}{n} \times D$$

5. Consumer Credit

i. The effective rate of interest is the discount rate in the equation:

Loan Amount – P.V (Installments paid) – Service Fee + P.V (Accumulated Value of Deposit) + P.V (Prompt Payment Bonus) = 0

6. Housing Finance

i. Disbursement Amount, RD

$$= AV \times \frac{CC}{100} \times \frac{PC}{100} + AV \times \frac{LC}{100} - BC - CM$$

Where,

RD = Recommendation for disbursement in rupees

AV = Aggregate value = LC + CC

PC = Progress of construction in % points

LC = Land component

CC = Cost of construction + Overheads + Profits

BC = Borrower's contribution

CM = Cumulative disbursement made

ii. Equated Monthly Installments $= \frac{1}{12}\left(\frac{Lr(1+r)^n}{(1+r)^n - 1}\right)$

Where,

L = Loan

r = Rate of interest in decimals

n = Period.

7. Venture Capital

i. NPV = [(Cash) / (Post)] × [(PAT × PER)] × k

Where,

NPV = Net Present Value of the cash flows relating to the investment

Post = Pre + cash

Cash represents the amount of cash.

'pre' = The pre-money valuation of the firm estimated by the 'investor'

k = The PVIF for the investment horizon

PER = Price Earnings Multiple

PAT = Profit After Tax.

VII. Management Accounting

1. Cost-Volume-Profit Analysis

i. Break-Even Point (Units) $= \dfrac{\text{Fixed Cost}}{\text{Selling Price per Unit} - \text{Variable Cost per Unit}}$

$$= \frac{\text{Fixed Cost}}{\text{Contribution per Unit}}$$

Or, $\dfrac{\text{Break Even Sales(Rs.)}}{\text{Selling Price per Unit}}$

ii. Break-Even Point (Rs.)

$$= \frac{\text{Fixed Cost x Selling Price per Unit}}{\text{Selling Price per Unit} - \text{Variable Cost per Unit}}$$

$$= \frac{\text{Fixed Cost x Selling Price per Unit}}{\text{Contribution per Unit}}$$

$$= \frac{\text{Fixed Cost}}{\text{Contribution per unit} \div \text{Selling price per unit}}$$

$$= \frac{\text{Fixed Cost}}{\text{P / V ratio}} = \frac{\text{Fixed Cost}}{1 - \dfrac{\text{Variable Cost}}{\text{Sales}}}$$

Or,

Break-even Point (Units) × Selling Price per Unit

iii. At Break-even Point

Sales – Variable Cost – Fixed Cost = 0

Or, Contribution – Fixed Cost = 0

Or, Contribution = Fixed Cost

iv. Calculation of Required Sales value to earn a desired amount of profit

$$= \frac{\text{Fixed Cost} + \text{Desired Profit}}{\text{P/V Ratio}}$$

v. Profit/Volume Ratio

a. P/V Ratio $= \dfrac{\text{Sales} - \text{Variable Cost}}{\text{Sales}} \text{x} 100$

$$= \frac{\text{Contribution}}{\text{Sales}} \text{x} 100$$

$$= \frac{\text{Fixed Cost} + \text{Profit}}{\text{Sales}} \text{x} 100$$

Or,

$$\frac{\text{Selling price per unit} - \text{Variable cost per unit}}{\text{Selling price per unit}} \text{ x } 100$$

$$= \frac{\text{Contribution per unit}}{\text{Selling price per unit}} \text{ x } 100$$

b. P/V ratio $= \frac{\text{Change in Contribution}}{\text{Change in Sales}} \text{x}100$

Or

$\frac{\text{Change in Contribution per unit}}{\text{Change in Selling Price per unit}} \text{ x } 100$

Or

$\frac{\text{Change in Profit}}{\text{Change in Sales}} \text{x}100$

vi. Margin of Safety = Total Sales – Break-even Sales

Or $\frac{\text{Profit}}{\text{P / V ratio}}$

$= \frac{\text{Profit} \times \text{Selling price per unit}}{\text{Selling price per unit} - \text{Variable cost per unit}}$

vii. Margin of Safety as a percentage of Total Sales

$= \frac{\text{Margin of Safety}}{\text{Total Sales}} \text{ x } 100$

2. Standard Costing and Variance Analysis

i. Material Cost Variance

= Usage Variance + Price Variance

ii. Material Cost Variance = (SQ × SP) – (AQ × AP)

iii. Material Usage Variance = (SQ – AQ) × SP

iv. Material Price Variance = (SP – AP) × AQ

Where,

SQ = Standard Quantity for the actual output

SP = Standard Price

AQ = Actual Quantity

AP = Actual Price

v. Material Mix Variance = (Standard cost of standard mix of the actual quantity – Standard cost of actual mix of the actual quantity)

Or

= (Revised standard mix of actual input – Actual mix) × Standard Price

vi. Material Yield Variance = (Standard yield specified – Actual yield) × Standard cost per unit

Or

= (Standard loss on actual input – Actual loss) × Standard cost per unit

vii. Sub-usage Variance = (Standard quantity – Revised standard proportion of actual input) × Standard cost perunit of input

viii. Labor Cost Variance = Efficiency Variance + Rate Variance

ix. Labor Cost Variance = (SH × SR) – (AH × AR)

Where,

SH = Standard Hours

SR = Standard Rate

AH = Actual Hours

AR = Actual Rate

x. Labor Efficiency Variance = (Standard hours for the actual output – Actual hours) × Standard rate per hour

xi. Labor Rate Variance = (Standard rate – Actual rate) × Actual hours

xii. Labor Mix Variance = (Revised standard labor mix in terms of actual total hours – Actual labor mix) × Standard rate per hour

xiii. Labor Yield Variance = (Standard output based on actual hours – Actual output) × Average standard labor rate per unit of output

Or

= (Standard loss on actual hours – Actual loss) × Average standard labor rate per unit of output

xiv. Labor Efficiency Sub-variance = (Standard mix – Revised Standard mix) × Standard rate.

VIII. Portfolio Management

1. Capital Market Theory

i. Variance of a portfolio of n securities: $\sigma_n^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} W_i W_j \sigma_{ij}$

Where,

W_i = Weight of ith security

W_j = Weight of jth security

σ_{ij} = Covariance between ith security and jth security

When portfolios are equally weighted,
the expected level of portfolio risk can be expressed as

$$E\left(\sigma_n^2\right) = 1/n\left[E\left(\sigma_i^2\right) - E\left(\sigma_{ij}\right)\right] + E\left(\sigma_{ij}\right)$$

Where,

$E\left(\sigma_i^2\right)$ = Average variance of an individual security that is included in the portfolio

$E\left(\sigma_{ij}\right)$ = Average pair wise covariance between securities in the portfolio

ii. Tax-adjusted CAPM $E(r_i)$

$= r_f (1 - T) + \beta_i [E(r_M) - r_f (1 - T) - TD_m] + TD_i$

Where,

$E(r_i)$ = Expected return on stock i

r_f = Risk-free rate of interest

β_i = Beta coefficient of stock i

D_m = Dividend yield on the market portfolio

D_i = Dividend yield on stock i

$T = \dfrac{\left(T_d - T_g\right)}{\left(1 - T_g\right)}$ = Tax factor

T_d = Tax rate on dividends

T_g = Tax rate on (long-term) capital gains.

2. Arbitrage Pricing Theory (APT Model)

i. $E(r_i) = \tau_0 + \tau_1\beta_{i1} + \tau_2\beta_{i2} + \tau_3\beta_{i3} + ... + \tau_M\beta_{iM}$

Where,

$E(r_i)$ = Expected return on Asset i

τ_0 = Expected return on an asset with zero systematic risk

= r_f if riskless borrowing and lending exist

τ_j = Risk premium, or market price of risk, associated with the jth factor

= $E(r_j) - \tau_0$, or τ_j riskless borrowing and lending exist

β_{ij} = Sensitivity or beta coefficient for security i that is associated with index j.

3. Asset Allocation

i. $U_{mk} = E(R_m) - \dfrac{\sigma_m^2}{t_k}$

Where,

U_{mk} = The expected utility of asset mix m for investor k

$E(R_m)$ = The expected return for asset mix m

σ_m^2 = The standard deviation for asset mix m

t_k = Investor k's risk tolerance.

4. Delineating Efficient Frontiers

i. Optimal portfolio selection using sharpe's optimization

a. Cut-off point $(C_i) = \dfrac{\sigma_M^2 \sum_{i=1}^{i} \frac{(R_i - R_f)}{\sigma_{ei}^2}\beta_i}{1+\sigma_M^2 \sum_{i=1}^{i} \frac{\beta_i^2}{\sigma_{ei}^2}}$

b. The proportion invested in each security, $X_i = \dfrac{Z_i}{\sum_{j=1}^{N} Z_j}$

c. The relative investment in each security

$$Z_i = \frac{\beta_i}{\sigma_{ei}^2}\left[\frac{R_i - R_f}{\beta_i} - C^*\right]$$

Where,

σ_M^2 = Variance in the market index

σ_{ei}^2 = The stock unsystematic risk

R_i = Expected return on stock i

R_f = Risk-free rate of return

β = Beta of the stock.

5. Portfolio Analysis

i. Expected return of a portfolio of n securities, $E_p = \sum_{i=1}^{n} W_i\, E(R_i)$

Where,

E_p = The portfolio return

W_i = The proportion of investment in security i

$E(R_i)$ = The expected return on security i

n = The total number of securities in the portfolio

ii. Holding period yield $= \dfrac{(P_{it} - P_{it-1}) + D_t}{P_{it-1}}$

Where,

P_{it} = The current price of the security

P_{it-1} = The price of the security at the beginning of period t

D_t = The dividend received during the period t

iii. Variance or Risk of a portfolio

$$Var(R_p) = \sum_{i=1}^{n} W_i^2 \, Var(R_i) + \sum_{j=1}^{n} \sum_{i=1,\, i \neq j}^{n} W_i \, W_j \, Cov(R_i \, R_j)$$

Where,

$Var(R_p)$ = The variance of the return on the Portfolio

$Var(R_i)$ = Variance of return on security i

$Cov(R_i \, R_j)$ = The covariance between the returns of securities i and j

W_i, W_j = The percentage of investable funds invested in securities i and j.

iv. Correlation Co-efficient, $\rho_{ij} = \dfrac{\sigma_{ij}}{\sigma_i \, \sigma_j}$

Where,

σ_{ij} = Covariance between securities i and j

σ_i = Standard deviation of security i

σ_j = Standard deviation of security j

v. Systematic risk of security i = $\beta_{im}^2 \, \sigma_m^2$

Where,

β_{im}^2 = The beta of the security i

σ_m^2 = The variance of the market portfolio

vi. Systematic risk of the portfolio = $\left(\sum_{i=1}^{n} X_i \, \beta_{im} \right)^2 \sigma_m^2$

Where,

X_i = Proportion of the total portfolio invested in security i

n = Total number of stocks

β_{im}^2 = The beta of the security i

σ_m^2 = The variance of the market portfolio

vii. Unsystematic risk of portfolio = $\sum_{i=1}^{n} X_i^2 \, \sigma_{ei}^2$

Where,

X_i = Proportion of the total portfolio invested in security i

n = Total number of stocks

σ_{ei}^2 = Variance in security not caused by its relationship to the index

viii. Total portfolio variance (risk), $\sigma_p^2 = \left(\sum_{i=1}^{n} X_i \, \beta_{im} \right)^2 \sigma_m^2 + \left(\sum_{i=1}^{n} X_i^2 \, \sigma_{ei}^2 \right)$

Where,

σ_p^2 = Variance of portfolio return

σ_m^2 = Expected variance of index

σ_{ei}^2 = Variance in security not caused by its relationship to the index

X_i = Proportion of the total portfolio invested in security i

n = Total number of stocks.

6. Portfolio Performance

i. Jensen's differential return (α_i) = $R_i - [R_f + \beta_i (R_m - R_f)$

Where,

R_i = Average realized return on portfolio P

R_f = Risk-free return for period t

R_m = Average return of the market portfolio for period t

β_i = A measure of systematic or market risk.(slope of the regression equation)

α_i = Intercept that measures the forecasting ability of the portfolio manager

ii. Treynor's ratio = $\frac{(R_P - R_f)}{\beta_p}$

Where,

R_P = Return on the portfolio

R_f = Risk-free rate of return

β_p = Beta of the portfolio

iii. Sharpe's ratio = $\frac{R_P - R_f}{\sigma_P}$

Where,

R_P = Return on the portfolio

R_f = Risk-free rate of return

σ_P = Standard deviation of return on the portfolio

iv. Return from total selectivity

= Return from net selectivity + Return for extra diversifiable risk

(or)

Return from net selectivity

= Return from total selectivity – Return for extra diversifiable risk

v. Return from net selectivity = $R_P - \left[R_f + (R_m - R_f)\frac{\sigma_P}{\sigma_m}\right]$

Where,

R_p = Return on portfolio

σ_p = Standard deviation of returns of portfolio p

σ_m = Standard deviation of market returns

R_f = Risk-free rate

R_m = Return on market portfolio.

7. Bond Portfolio Management

i. Number of futures contracts, $X = \frac{(D_T - D_I) P_I}{D_F P_F}$

Where,

X	=	Approximate number of futures contracts
D_T	=	Target effective duration for the portfolio
D_I	=	Initial effective duration for the portfolio
D_F	=	Effective duration for the futures contract
P_I	=	Initial market value of the portfolio
P_F	=	Market value of the futures contract.

IX. Project Management

1. Appraisal Criteria

i. Cash flow as per long-term funds point of view

= PAT + Depreciation + Interest on long-term (1 – t)

ii. Cash flow as per equity funds point of view

= PAT + Depreciation – Repayment of long-term borrowings – Repayment of short-term bank borrowings

iii. Modified Net Present Value

$$NPV_n = \frac{TV}{(1+k)^n} - I$$

Where,

NPV_n = Modified net present value

TV = Terminal value

k = Cost of capital

I = Investment outlay

$$TV = \sum_{t=1}^{n} CF_t (1+r')^{n-t}$$

n = Project life

Where,

CF_t = Cash in flow at the end of the year t

r' = Reinvestment rate applicable to the cash inflows of the project

iv. Modified Internal Rate of Return

$$r^* = \left[\frac{TV}{I}\right]^{1/n} - I$$

Where,

I = Initial investment

r^* = Modified IRR

n = Project life

TV = Terminal value

$I(1 + r^*)^n$ = TV.

2. Risk Analysis in Capital Investment Decisions

i. Expected NPV and Standard Deviation of NPV

a. In perfectly correlated cash flows

$$\text{Expected NPV}\left(\overline{NPV}\right) = \sum_{t=1}^{n} \bar{A}_t / (1+i)^t - I$$

$$\text{S.D. of the NPV} = \sum_{t=1}^{n} \sigma_t / (1+i)^t$$

b. In uncorrelated cash flows

$$\text{Expected NPV}\left(\overline{NPV}\right) = \sum_{t=1}^{n} \bar{A}_t / (1+i)^t - I$$

S.D. of the NPV $= \left[\sum_{t=1}^{n} \sigma_t^2 / (1+i)^{2t}\right]^{1/2}$

Where,

$\bar{A}_t$ = The expected cash flows for a time period t

i = The risk-free discount rate

n = The life of the project

$\overline{NPV}$ = The expected net present value

σ_t = Standard deviation of the cash flows for a time period t

I = Initial investment.

3. Application of Portfolio Theories in Investment Risk Appraisal

i. Asset beta, $\beta_A = \beta_E\left(\frac{E}{E+D}\right) + \beta_D\left(\frac{D}{E+D}\right)$

Where,

β_A = Asset beta

β_E = Equity beta

β_D = Debt beta.

4. Social Cost Benefit Analysis

i. Effective Rate of Protection

$$= \frac{\text{Value added at domestic prices} - \text{Value added at world prices}}{\text{Value added at world prices}}$$

ii. Domestic resource cost = $\frac{\text{Value added at domestic prices} \times \text{Exchange rate}}{\text{Value added at world prices}}$

5. Options in Investment Appraisal

i. Transaction price or cash price of the bond,

P = Quoted price + Accrued interest

Invoice price = (Futures settlement price × Conversion factor) + Accrued interest

ii. $HR = -\left(\frac{\text{Cash market principal}}{\text{Futures market principal}}\right) \times \text{Conversion factor}$

iii. $HR = \left(\frac{\text{Cash flow to be hedged}}{\text{Value of futures contract}} \times \text{Conversion factor} \times \frac{\text{Portfolio duration}}{\text{CTD bond duration}}\right)$

iv. Binomial Pricing Model

Call price, $C = \frac{C_u p + C_d(1-p)}{R}$

$p = \frac{R-d}{u-d}$

Where,

u = 1 + percentage increase in stock price from time 0 to time t

d = 1+ percentage decrease in stock price from time 0 to time t

C = The call price

C_u = The value of the call if the stock price increases

C_d = The value of call if the stock price decreases

R = 1+ risk-free rate of return (r)

p = Probability of price increase

v. Black-Scholes option pricing model:

$C = S_0N(d_1) - Xe^{-r(T-t)}N(d_2)$

$P = Xe^{-r(T-t)}N(-d_2) - S_0N(-d_1)$

Where,

$$d_1 = \frac{\ln(S_0/X) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{(T-t)}}$$

$$d_2 = \frac{\ln(S_0/X) + \left(r - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{(T-t)}}$$

Or,

d_2 = $d_1 - \sigma\sqrt{T-t}$

C = The call option price

P = The put option price

S = The spot price of the underlying asset

X = The strike price of the option

r = The risk-free rate

(T – t) = The time to expiration expressed in years

σ = The annualized standard deviation of returns on the underlying asset, i.e., the volatility measure

N(d) = Cumulative standard normal distribution

e = Exponential function

In = Natural logarithm.

6. Project Scheduling

i. Expected time (t_e) = $\frac{t_o + 4t_m + t_p}{6}$

ii. Variance (V) = $\left[\frac{t_p - t_o}{6}\right]^2$

Where,

t_o = Optimistic estimate of time

t_m = Most likely time

t_p = Pessimistic estimate of time.

7. Project Monitoring and Control

i. Cost Performance Index = $\frac{BCWP}{ACWP}$

ii. Schedule Performance Index = $\frac{BCWP}{BCWS}$

iii. Estimated Cost Performance Index $= \dfrac{\text{BCTW}}{\text{ACWP} + \text{ACC}}$

Where,

BCWP	=	Budgeted cost of work performed
ACWP	=	Actual cost of work performed
BCWS	=	Budgeted cost of work scheduled
BCTW	=	Budgeted cost for total work
ACC	=	Additional cost for completion.

X. Quantitative Methods

1. Basics of Mathematics

i. Progressions

a. The nth term in an A.P.

$T_n = a + (n - 1)d$

b. Sum of all the terms in an A.P.

$S = \frac{n}{2}\{2a + (n - 1)d\}$

Where,

a = First term

n = No. of terms

d = Common difference

c. The nth term in a G.P.

$T_n = ar^{n-1}$

d. Sum of numbers in a G.P.

$S = \frac{a(r^n - 1)}{(r - 1)} \quad r \neq 1$

e. Sum of an Infinite G. P.

$S = \frac{a}{1 - r}$

Where,

a = First term

r = Common ratio

ii. Permutations and Combinations

a. Permutations

${}^nP_r = \frac{n!}{(n - r)!}$

Where,

n! = n(n – 1) (n – 2) (n – 3) 3.2.1

b. Combinations

${}^nC_r = \frac{{}^nP_r}{r!} = \frac{n!}{(n - r)!r!}$

iii. Logarithms

a. $\log_a MN = \log_a M + \log_a N$

b. $\log_a (M/N) = \log_a M - \log_a N$

c. $\log_a (M^p) = p.\log_a M$

d. $\log_b a \times \log_a b = 1$

2. Calculus

i. Rules of Differentiation

a. If $f(x) = x^n$

then $f'(x) = nx^{n-1}$

b. If $f(x) = g(x)\,h(x)$

then $f'(x) = g'(x)\,h(x) + g(x)\,h'(x)$

c. If $f(x) = \frac{g(x)}{h(x)}$ where $h(x) \neq 0$

then $f'(x) = \frac{g'(x)h(x) - g(x)h'(x)}{[h(x)]^2}$

d. If f(x) = c. g(x) where 'c' is a constant,

then $f'(x) = c\,g'(x)$

e. If f(x) = g(x) + h(x)

then $f'(x) = g'(x) + h'(x)$

f. If f(x) = ln x, then $f'(x) = \frac{1}{x}$

If $f(x) = e^{g(x)}$

then $f'(x) = g'(x).e^{g(x)}$

g. If f(x) = ln g(x),

then $f'(x) = \frac{g'(x)}{g(x)}$

h. $f^n(x) = \frac{d^n f(x)}{dx^n}$

ii. Partial Derivatives

a. For a function, f = g(x,y) . h(x,y)

$$\frac{\partial f}{\partial x} = g(x,y)\frac{\partial h}{\partial x} + h(x,y)\frac{\partial g}{\partial x}$$

$$\frac{\partial f}{\partial y} = g(x,y)\frac{\partial h}{\partial y} + h(x,y)\frac{\partial g}{\partial y}$$

b. For a function, $f = \frac{g(x,y)}{h(x,y)}$ and h (x,y) ≠0,

$$\frac{\partial f}{\partial x} = \frac{h(x,y)[\partial g/\partial x] - g(x,y)[\partial h/\partial x]}{[h(x,y)]^2}$$

$$\frac{\partial f}{\partial y} = \frac{h(x,y)[\partial g/\partial y] - g(x,y)[\partial h/\partial y]}{[h(x,y)]^2}$$

c. For a function, $f = [g(x,y)]^n$

$$\frac{\partial f}{\partial x} = n[g(x,y)]^{n-1}\frac{\partial g}{\partial x}$$

$$\frac{\partial f}{\partial y} = n[g(x,y)]^{n-1}\frac{\partial g}{\partial y}$$

iii. Integration

a. If 'K' and 'c' are constants, then

$$\int K dx = Kx + c$$

b. $\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$

c. $\int x^{-1}dx = \ln x + c, x > 0$

d. $\int a^{Kx}.dx = \frac{a^{Kx}}{K\ln a} + c$, where 'a' and 'K' are constants

e. $\int e^{Kx}.dx = \frac{e^{Kx}}{K} + c$

f. $\int Kf(x)dx = K\int f(x)dx$

g $\int [f(x)+g(x)]dx = \int f(x)dx + \int g(x)dx$

h. $\int [-f(x)dx] = -\int [f(x)dx]$

iv. Definite Integral

a. $\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$

Where, F(x) is the indefinite integral of f(x)

b. $\int_c^d f(x)dx = -\int_d^c f(x)dx$

c. $\int_k^k f(x)dx = F(k) - F(k) = 0$

d. $\int_p^r f(x)dx = \int_p^q f(x)dx + \int_q^r f(x)dx$

Where, $p \le q \le r$

e. $\int_c^d f(x)dx \pm \int_c^d g(x)dx = \int_c^d [f(x) \pm g(x)]dx$

f. $\int_q^r cf(x)dx = c\int_q^r f(x)dx$

Where, c is a constant.

3. Interpolation and Extrapolation

i. Linear Approximation Method of Interpolation

Interpolated figure

a. For ascending series:

$$= \text{Base value} + \frac{\text{Upper limit} - \text{Lower limit}}{(t_s - t_p)} \times (t_i - t_p)$$

b. For descending series:

$$= \text{Base value} - \frac{\text{Lower limit} - \text{Upper limit}}{(t_s - t_p)} \times (t_i - t_p)$$

Where,

Base value is the value of the immediately preceding year.

$(t_i - t_p)$: time interval between the immediately preceding year and the year for which the value is to be interpolated.

$(t_s - t_p)$: time interval between the two known values.

4. Central Tendency and Dispersion

i. Arithmetic Mean

$$\overline{x} = (x_1 + x_2 + ... + x_n)/n = \frac{\left(\sum_{i=1}^{n} x_i\right)}{n}$$

Where,

n = No. of observations

ii. Mean for discrete series or ungrouped data

$$\overline{x} = \frac{\sum_{i=1}^{k} f_i x_i}{\sum_{i=1}^{n} f_i}$$

Where,

f = Frequency

iii. Mean for continuous series or grouped data, $\overline{x} = \frac{\sum fm}{N}$

Where,

m = Midpoint of class

$= \frac{\text{Lower limit} + \text{Lower limit of next class}}{2}$

f = Frequency of each class

N = $\sum f$ = Total frequency

iv. Weighted arithmetic mean

$$= \overline{x}_W = \frac{\sum WX}{\sum W}$$

v. Median

a. For ungrouped data,

If the total of the frequencies is odd, say n

Median = Value of $\frac{(n+1)}{2}^{th}$ item

If total of the frequencies is even, say 2n

Median = Arithmetic Mean of nth and (n + 1)th items

b. For grouped data,

$$\text{Median} = \left[\frac{(N+1)/2 - (F+1)}{f_m}\right] w + L_m$$

Where,

L_m = Lower limit of the median class

f_m = Frequency of the median class

F = Cumulative frequency up to the lower limit of the median class

w = Width of the class interval

N = Total frequency

vi. Mode (For a grouped data)

$$\text{Mode} = L_{mo} + \frac{f_{mo} - f_1}{2f_{mo} - f_1 - f_2} \times w$$

Where,

L_{mo} = Lower limit of the modal class which is the class having the maximum frequency

f_1, f_2 = Frequencies of the classes preceding and succeeding the modal class respectively

f_{mo} = Frequency of modal class

w = Class interval

vii. Empirical mode = 3Median – 2Mean

viii. Geometric mean, $G = (X_1 \times X_2 \times X_3 \ldots X_n)^{1/n}$

ix. Harmonic mean, $HM = \dfrac{N}{\frac{1}{x_1} + \frac{1}{x_2} + \ldots + \frac{1}{x_n}}$

x. Weighted Harmonic mean, $WHM = \dfrac{\sum w}{\sum (w/x)}$

xi. Mean Absolute Deviation = $\dfrac{\sum |x - \bar{x}|}{n}$

Here, $|x - \bar{x}| = x - \bar{x}$ if $x \geq \bar{x}$

and $|x - \bar{x}| = \bar{x} - x$ if $x \leq \bar{x}$

xii. Quartile Deviation Q.D. = $\dfrac{Q_3 - Q_1}{2}$

Where,

Q_1 = First quartile = Size of $\frac{N}{4}$ th observation

Q_3 = Third quartile = Size of $\frac{3N}{4}$ th observation

N = Number of observations

xiii. Population standard deviation

$$\sigma = \sqrt{\frac{\sum (x - \mu)^2}{N}}$$

Where,

x denotes each observation

μ = Arithmetic mean of population

N = No. of observations

For grouped data,

$$\sigma = \sqrt{\frac{\sum f (x - \mu)^2}{\sum f}}$$

Where,

f = Frequency

μ = Arithmetic mean of population

xiv. Sample standard deviation

$$S = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

Where,

$\bar{x}$ = Sample mean

xv. Combined standard deviation of two groups

$$\sigma_{12} = \sqrt{\frac{N_1\sigma_1^2 + N_2\sigma_2^2 + N_1 d_1^2 + N_2 d_2^2}{N_1 + N_2}}$$

Where,

μ_1 = Mean of first group

μ_2 = Mean of second group

σ_1 = Standard deviation of first group

σ_2 = Standard deviation of second group

N_1 = Number of observations in the first group

N_2 = Number of observations in the second group

d_1 = $\mu_1 - \mu$

d_2 = $\mu_2 - \mu$

μ = $\frac{(N_1\mu_1 + N_2\mu_2)}{N_1 + N_2}$

xvi. Standard Deviation of a Discrete Random Variable $\sigma = \left[\sum_{i=1}^{n} P_1 \left(k_i - \bar{k}\right)^2\right]^{1/2}$

Where,

P_i = Probability associated with the occurrence of the ith value

k_i = ith possible value

k = Expected rate of return i.e. mean

n = Number of possible outcomes

xvii. Coefficient of variation = $\frac{\text{Standard Deviation}}{\text{Mean}} \times 100$

5. Probability

i. Marginal or unconditional probability of an event A

$$P(A) = \frac{\text{Number of possible outcomes favoring A}}{\text{Total number of possible outcomes}}$$

ii. If A and B are mutually exclusive events,
then P(A or B) = P(A) + P(B)

iii. If A and B are not mutually exclusive,
P(A or B) = P(A) + P(B) – P(A and B)

iv. If A and B are independent events,
P(A and B) = P(A) . P(B)

v. Conditional probability of event A, given that B has occurred, in case of A and B being independent events is P(A/B) = P(A)

vi. If A and B are dependent then

P(A and B) = P(A) . P(B/A)

or P(B and A) = P(B) . P(A/B)

vii. Bayes' Theorem:

$$P(A_i/B) = \frac{P(A_i)P(B/A_i)}{P(A_1)P(B/A_1)+P(A_2)P(B/A_2)+...+P(A_k)P(B/A_k)}$$

6. Probability Distribution and Decision Theory

i. Expected Value

$E[x] = \sum x\,P(x)$,

Where,

x = Random variable

P(x) = Probability of x

ii. Covariance

a. For a population of paired ungrouped data points {x,y}

$$Cov_{xy} = \frac{\Sigma(x-\mu_x)(y-\mu_y)}{N}$$

Where,

μ_x = The arithmetic mean of {x}

μ_y = The arithmetic mean of {y}

N = The number of observations in each population

For a paired sample {x,y},

$$Cov_{xy} = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{n-1}$$

Where,

$\bar{x}$ = The arithmetic mean of sample {x}

$\bar{y}$ = The arithmetic mean of sample {y}

b. For grouped data of paired population

$$Cov_{xy} = \frac{\Sigma f\,(x-\mu_x)(y-\mu_y)}{\Sigma f}$$

Where,

f = The frequency of the corresponding (x,y) values.

Given a probability distribution of paired data {x,y},

$$Cov_{xy} = \Sigma\left[x-E(x)\right]\left[y-E(y)\right]P(x,y)$$

Where,

P(x,y) = The joint probability of x and y

E(x) = The expected value of x

E(y) = The expected value of y.

iii. $E(a_1X_1 + a_2X_2) = a_1E(X_1) + a_2E(X_2)$

iv. $V(a_1 X_1 + a_2 X_2) = a_1^2 V(X_1) + a_2^2 V(X_2) + 2a_1a_2 \text{Cov}(X_1, X_2)$

Where, V denotes variance

v. Binomial distribution,

$$f(x) = \binom{n}{x} p^x (1-p)^{(n-x)}$$

Where,

f(x) = The probability of x successes in n trials

n = The number of trials

$\binom{n}{x} = \frac{n!}{x!(n-x)!}$

p = The probability of a success on any one trial

(1 – p) = q = The probability of a failure on any one trial

E(x) = np

V(x) = npq

vi. Poisson Distribution

$$f(x) = \frac{\lambda^x \times e^{-\lambda}}{x!}$$

Where,

f(x) = Probability of x occurrences in an interval

λ = The mean number of occurrences in an interval

e = The base of natural logarithm system

vii. Hypergeometric distribution

$$f(x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}} \quad \text{for } 0 \le x \le r$$

Where,

f(x) = Probability of x successes in n trials

n = Number of trials

N = Number of elements in the population

r = Number of elements in the population labeled success

$E(x) = \frac{nr}{N}$

$V(x) = \frac{nr(N-r)(N-n)}{N^2(N-1)}$

viii. Standard Normal Variable,

$$z = \frac{x-\mu}{\sigma}$$

Where,

x = Random variable

μ = Mean of the distribution of the random variable

σ = Standard deviation

ix. In a t-distribution, $t = \frac{\overline{x}-\mu}{S/\sqrt{n}}$

Where,

$\overline{x}$	=	The sample mean
μ	=	The population mean
S	=	Sample standard deviation
n	=	The sample size.

x. If MP = Marginal profit

ML = Marginal loss

'P' = The probability of generating the additional profit by increasing our activity level by one unit, then

Expected (MP) = P × MP

Expected (ML) = (1 – P) × ML

$$P^* = \frac{ML}{ML + MP}$$

P* represents the minimum required probability of selling at least one additional unit to justify the stocking of that additional unit.

7. Statistical Inferences

i. Standard Error for

a. Sample mean ($\overline{x}$), $\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$

b. Sample proportion ($\overline{p}$), $\sigma_{\overline{p}} = \sqrt{\frac{pq}{n}}$

Where, q = 1 – p

c. Difference of two sample means $\overline{x}_1$ and $\overline{x}_2$ i.e.

$$\sigma_{\overline{x}_1-\overline{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Where,

$\overline{x}_1$ and $\overline{x}_2$ are the means of two random samples of sizes and drawn from two populations with standard deviations σ_1 and σ_2 respectively.

d. Difference of two proportions:

$$\sigma_{\overline{P}_1-\overline{P}_2} = \sqrt{\frac{\hat{p}\hat{q}}{n_1} + \frac{\hat{p}\hat{q}}{n_2}}$$

Where,

$\overline{p}_1$ and $\overline{p}_2$ are the proportions of two random samples of sizes n_1 and n_2 drawn from two populations and $\hat{p} = \frac{n_1\overline{p}_1 + n_2\overline{p}_2}{n_1 + n_2}$ and $\hat{q} = 1 - \hat{p}$

Where,

$\hat{p}$ is the estimate of the overall proportion of success in the combined populations using combined proportions for both the samples.

e. For a finite population of size N, when a sample of size n is drawn without replacement,

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$$

f. Sample standard deviation,

$$\sigma_s = \sqrt{\frac{\sigma^2}{2n}}$$

8. Simple Linear Regression and Correlation

i. Karl Pearson's correlation coefficient,

$$r = \frac{Cov(X,Y)}{s_x s_y}$$

Or

$$r = \frac{\Sigma(X-\bar{X})(Y-\bar{Y})}{\sqrt{\Sigma(X-\bar{X})^2\,\Sigma(Y-\bar{Y})^2}}$$

ii. Rank correlation coefficient,

$$R = 1 - \frac{6\sum_{i=1}^{n} D_i^2}{n^3 - n}$$

Where, D_i = The difference in the ranks of the ith individual

iii. If the regression line, $\hat{Y}_X = a + bX$

$$b = \frac{n\sum XY - (\sum X)(\sum Y)}{n\sum X^2 - (\sum X)^2}$$

$$a = \bar{Y} - b\bar{X}$$

iv. Standard error of the estimate for a simple regression equation

$$S_e = \sqrt{\frac{\Sigma\left(Y-\hat{Y}\right)^2}{n-2}} = \sqrt{\frac{\Sigma Y^2 - a\Sigma Y - b\Sigma XY}{n-2}}$$

Where,

Y = Values of dependent variable

$\hat{Y}$ = Estimated values from the estimating equation that correspond to each Y value

n = Number of data points used to fit the regression line

v. Total Sum of Squares, TSS $= \Sigma\left(Y-\bar{Y}\right)^2$

Regression Sum of Squares, RSS $= \Sigma\left(\hat{Y}-\bar{Y}\right)^2$

Error Sum of Squares, ESS $= \Sigma\left(Y-\hat{Y}\right)^2$

vi. Coefficient of Determination, $R^2 = \dfrac{a\Sigma Y + b\Sigma XY - n\bar{Y}^2}{\Sigma Y^2 - n\bar{Y}^2}$

vii. $S_b = \dfrac{S_e}{\sqrt{\Sigma x^2 - n\bar{x}^2}}$

Where, S_b = Estimate of V(b).

9. Multiple Regression

i. Multiple Regression Equation

$\hat{Y} = A + B_1X_1 + B_2X_2 + B_3X_3 + \ldots.. + B_nX_k$

ii. Standard error of estimate for a multiple regression equation

$$S_e = \sqrt{\frac{\Sigma\left(Y - \hat{Y}\right)^2}{(n-k-1)}}$$

$$= \sqrt{\frac{\Sigma Y^2 - a\Sigma Y - b_1\Sigma X_1 Y - b_2\Sigma X_2 Y}{n-k-1}}$$

Where,

Y = The sample value of the dependent variable

= The corresponding estimate obtained by using the regression equation

n = number of observations

k = number of independent variables

iii. Coefficient of multiple correlation between Y and both X_1 and X_2 is given by

$$R_{Y.X_1X_2} = 1 - \sqrt{\frac{\left(\Sigma Y^2 - a\Sigma Y - b_1\Sigma X_1 Y - b_2\Sigma X_2 Y\right)}{\left(\Sigma Y^2 - (\Sigma Y)^2 / n\right)}}$$

$$R_{1.23} = \sqrt{\frac{r_{12}^2 + r_{13}^2 - 2r_{12}\, r_{23}\, r_{13}}{1 - r_{23}^2}}$$

iv. In the equation, $Y = a + b_1X_1 + b_2X_2$,

the partial correlation coefficient is given by R_{123}

Where,

$R_{123} = \dfrac{r_{12} - r_{13}.r_{23}}{\sqrt{\left(1 - r_{13}^{\,2}\right)\left(1 - r_{23}^{\,2}\right)}}$ is the partial correlation coefficient between Y and X_1, when X_2 is kept constant.

Where,

r_{12} = Correlation coefficient between Y and X_1

r_{23} = Correlation coefficient between X_1 and X_2

r_{13} = Correlation coefficient between Y and X_2

R_{123} will take values between 0 and 1, i.e., $0 \le R_{123} \le 1$.

10. Time Series Analysis

i. Secular Trend

Using regression analysis, estimating equation is

$\hat{Y} = a + bX$ (linear trend)

After coding (or translating time)

$a = \bar{Y}\ b = \dfrac{\Sigma xY}{\Sigma x^2}$

Where,

x = $(X - \bar{X})$ if there are odd number of data points and

x = $2\,(X - \bar{X})$ if there are even number of data points.

Curvilinear trend, $\hat{Y} = a + bX + X^2$

After coding (or translating time) is done

$\Sigma Y = an + c\Sigma x^2$

$\Sigma x^2 Y = a\Sigma x^2 + c\Sigma x^4$ and $b = \frac{\Sigma xY}{\Sigma x^2}$

Where,

x = $(X - \bar{X})$ if there are odd number of data points and

x = $2(X - \bar{X})$ if there are even number of data points

ii. Cyclical Variation

a. Percent of Trend Measure

Cyclical variation component $= \frac{Y}{\hat{Y}} \times 100$

Where,

Y represents actual values and

$\hat{Y}$ represents estimated values

b. Relative Cyclical Residual Measure

Cyclical Component $= \frac{Y - \hat{Y}}{\hat{Y}} \times 100$

11. Index Numbers

i. Unweighted Aggregates Price Index $= \frac{\Sigma P_1}{\Sigma P_0} \times 100$

Where,

ΣP_1 = Sum of all elements in the composite for current year

ΣP_0 = Sum of all elements in the composite for base year

ii. Weighted Aggregates Index

a. Laspeyre's Price Index

$= \frac{\Sigma P_1 Q_0}{\Sigma P_0 Q_0} \times 100$

b. Laspeyre's Quantity Index $= \frac{\Sigma Q_1 P_0}{\Sigma Q_0 P_0} \times 100$

Where,

P_1 = Prices in the current year

P_0 = Prices in the base year

Q_0 = Quantities in the base year

Q_1 = Quantities in the current year

c. Paasche's Price Index

$= \frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_1} \times 100$

d. Fisher's Ideal Price Index $= \sqrt{\frac{\Sigma P_1 Q_0}{\Sigma P_0 Q_0} \times \frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_1}} \times 100$

e. Fisher's Ideal Quantity Index $= \sqrt{\frac{\Sigma Q_1 P_0}{\Sigma Q_0 P_0} \times \frac{\Sigma Q_1 P_1}{\Sigma Q_0 P_1}} \times 100$

f. Marshall Edgeworth Price Index $= \frac{\Sigma(Q_0 + Q_1) P_1}{\Sigma(Q_0 + Q_1) P_0}$ x 100

iii. Value Index Number = $\frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_0}$ x 100

iv. Average of Relatives Method

a. Unweighted average of relatives method $= \frac{\Sigma\left(\frac{P_1}{P_0} \times 100\right)}{n}$

b. Unweighted average of relatives quantity index

$$= \frac{\Sigma\left(\frac{Q_1}{Q_0} \times 100\right)}{n}$$

c. Weighted average of relatives price index

$$= \frac{\Sigma\left[\left(\frac{P_1}{P_0} \times 100\right)(P_n Q_n)\right]}{\Sigma P_n Q_n}$$

Where,

P_n = Prices in the fixed period

Q_n = Quantities in the fixed period

$P_n Q_n$ = Value in the fixed period

v. Chain Index Numbers

Chain Index for a given year $= \frac{\text{Average link relative of the given year} \times \text{Chain index of previous year}}{100}$

Where,

Link relative $= \frac{\text{Price in a given period}}{\text{Previous year's price}}$ x 100

12. Quality Control

i. $\bar{X}$-Charts

a. When Mean and Standard Deviation are known:

Lower limit $= \mu_{\bar{x}} - 3\sigma_{\bar{x}}$

Upper limit $= \mu_{\bar{x}} + 3\sigma_{\bar{x}}$

b. When the mean and standard deviation are not known, then

Lower control limit = $\bar{\bar{X}} - A_2 \bar{R}$

Upper control limit = $\bar{\bar{X}} + A_2 \bar{R}$

Where,

$$\bar{\bar{X}} = \frac{1}{k}\Sigma\bar{X} = \frac{\Sigma x}{n \times k}$$

$$A_2 = \frac{3}{d_2 \sqrt{n}}$$

ii. R-Charts

Lower control limit = $D_3 \overline{R}$

Upper control limit = $D_4 \overline{R}$

Where,

$$D_3 = \left(1 - \frac{3d_3}{d_2}\right),\ D_4 = \left(1 + \frac{3d_3}{d_2}\right)$$

iii. p-Charts

When **p** is **known:**

Lower control limit = $p - 3\sigma_{\overline{p}}$

Upper control limit = $p + 3\sigma_{\overline{p}}$

Where,

$$\sigma_{\overline{p}} = \sqrt{\frac{pq}{n}}$$

When **p** is **unknown:**

$$\overline{\overline{p}} = \frac{\Sigma \overline{p}_j}{k}$$

Where,

$\overline{p}_j$ is the jth sample fraction

k is the number of all the samples considered

In calculating the lower and upper control limits $\overline{\overline{p}}$ is used instead of p.

13. Chi-Square Test and Analysis of Variance

i. The chi-square statistic is given by

$$\chi^2 = \sum \frac{(f_0 - f_e)^2}{f_e}$$

Where,

f_0 = The observed frequency

f_e = The expected frequency

ii. Number of Degrees of Freedom in a contingency table

= (Number of rows – 1) × (Number of columns – 1)

Number of degrees of freedom in chi-square test of goodness of fit for ‘k’ data points = k – 1

iii. ANOVA

a. Between-Column Variance, $\hat{\sigma}^2 = \dfrac{\Sigma n_j \left(\overline{x}_j - \overline{\overline{x}}\right)^2}{k-1}$

b. Within column variance, $\hat{\sigma}^2 = \Sigma \left(\dfrac{n_j - 1}{n_T - k}\right) S_j^2$

Where,

$\hat{\sigma}^2$ = The second estimate of the population variance

n_j = The size of the jth sample

n_T = The total number of elements present in all the samples

k = The number of samples

s_j^2 = The sample variance of the sample j

$\overline{x}_j$ = Mean of jth sample

$\overline{\overline{x}}$ = Grand mean

c. $$\text{F ratio} = \frac{\left(\begin{array}{c}\text{Population variance obtained from the variance among the}\\ \text{sample means (between column variance)}\end{array}\right)}{\left(\begin{array}{c}\text{Population variance obtained from the variance within}\\ \text{the individual samples (within column variance)}\end{array}\right)}$$

Degrees of freedom for the numerator = (k – 1)

DOF for Denominator = $\sum_{k=1}^{n}(n_j - 1) = n_T - k$

iv. Chi-square statistic for a sample variance is given by $\chi^2 = \frac{(n-1)s^2}{\sigma^2}$

Number of degrees of freedom = n – 1

v. Inferences about two population variances

F ratio for testing the equality of two population variances is given by

$$F = \frac{s_1^2}{s_2^2}$$

Where,

s_1^2 = The variance of the first sample

s_2^2 = The variance of the second sample

This distribution will have $n_1 - 1$ degrees of freedom in the numerator and $n_2 - 1$ degrees of freedom in the denominator respectively, n_1 and n_2 represent the number of elements present in each of the samples.

XI. Security Analysis

1. Bond Valuation

i. The intrinsic value or the present value of a bond

V_0 or $P_0 = I(PVIFA_{kd,\ n}) + F(PVIF_{kd,\ n})$

Where,

V_0 = Intrinsic value of the bond

P_0 = Present value of the bond

I = Annual interest payable on the bond

F = Principal amount (par value) repayable at the maturity time

n = Maturity period of the bond

k_d = Cost of Capital or Required rate of return

ii. Current yield = $\dfrac{\text{Coupon Interest}}{\text{Prevailing Market Price}}$

iii. Yield to maturity is r in the equation, $P_0 = \sum_{t=1}^{n} \dfrac{I}{(1+r)^t} + \dfrac{F}{(1+r)^n}$

Where,

P_0 = Present value of the bond

I = Annual interest payable on the bond

F = Principal amount (par value) repayable at the maturity time

n = Maturity period of the bond

iv. Realized yield is r in the equation $= P_0 (1 + r)^n$

$=$ Total cash flows received by the investor

v. Nominal Rate = Risk-free rate + Inflation rate

vi. Duration = $\dfrac{1C.PVIF_{r,1} + 2C.PVIF_{r,2} + ... + n[C+F]PVIF_{r,n}}{P_0}$

Where,

C = Coupon interest payments

r = Promised yield to maturity

n = Number of years to maturity

F = Redemption value

vii. Simplified formula for duration

$$D = \frac{r_c}{r_d} PVIFA_{(r_{d,n})} \text{ x } (1 + r_d) + \left[1 - \frac{r_c}{r_d}\right] n$$

Where,

r_c = Coupon yield

r_d = YTM

n = Number of years to maturity

viii. When bond is selling at par, (i.e. $r_c = r_d$)

Duration (D) = $PVIFA_{(r_{d,\ n})} \times (1 + r_d)$

Where,

r_c = Coupon yield

r_d = YTM

n = Number of years to maturity

ix. Duration of a perpetual bond, $D = \frac{1+r}{r}$

Where,

r = Current yield

x. Limiting value of duration $= \frac{1+YTM}{YTM}$

xi. Interest rate elasticity, $IE = \frac{\Delta P_0 / P_0}{\Delta YTM / YTM}$

Where,

ΔP_0 = Change in price for bond in period t

P_0 = Price of the bond at the period 0

ΔYTM = Change in YTM for the bond

YTM = Yield to maturity

xii. Approximate method of calculating interest rate elasticity

$$IE = D_{it} \times \frac{YTM}{1+YTM}$$

Where,

D_{it} = Duration

YTM = Yield to maturity

xiii. Interest rate risk which measures change in price of bond for a change in the YTM

$$\frac{\Delta P_0}{P_0} = IE_{it} \times \frac{\Delta YTM}{YTM}$$

Where,

IE_{it} = Interest rate elasticity

ΔP_0 = Change in price for bond in period t

P_0 = Price of the bond at the period 0

ΔYTM = Change in YTM for the bond

YTM = Yield to maturity

xiv. Modified Duration: $D_{mod} = \frac{D}{1+\frac{YTM}{f}}$

Where,

f = Discounting periods per year

D = Macaulay's duration

YTM = Yield to maturity in decimal form

xv. Percentage price volatility $= \frac{\Delta P}{P} \times 100 = -D_{mod}.\Delta y$

Where,

ΔP = Change in the price of the bond

P = Price of the bond

Δy = Change in YTM

D_{mod} = Modified duration

xvi. Duration of equity based on dividend discount model = $\frac{1}{k-g}$

Where,

k = Return required by equity holders

g = Constant growth rate of dividend

xvii. Duration of equity = $\frac{1}{\text{Dividend yield}} = \frac{\text{Market price}}{\text{Dividend}}$

2. Equity Stock Valuation Model

i. The intrinsic value or present value equity share

$$(P_0) = \sum_{t=1}^{n} \frac{D_t}{(1+k_e)^t} + \frac{P_n}{(1+k_e)^n}$$

Where,

P_0 = Current market price of the equity share or intrinsic value of the share

D_t = Expected equity dividend at time t

P_n = Expected price of the equity share at time n

k_e = Expected rate of return or required rate of return

ii. The value of equity share when there is constant growth

$$(P_0) = \frac{D_0(1+g)}{k_e - g}$$

Where,

P_0 = Intrinsic value of the share

D_0 = Current dividend per share

g = Expected constant growth rate in dividends

k_e = Expected rate of return or required rate of return

iii. The value of equity share using H Model

$$(P_0) = \frac{D_0\left[(1+g_n) + H(g_a - g_n)\right]}{r - g_n}$$

Where,

P_0 = Intrinsic value of the share

D_0 = Current dividend per share

r = Required rate of return

g_n = Normal long run growth rate

g_a = Current growth rate

H = One half of the period during which g_a will level off to g_n

3. Technical Analysis

i. Relative Strength, RS = $\left(\frac{\text{Average of 'x' days up-closings}}{\text{Average of 'x' days down-closings}}\right)$

ii. Relative strength index = 100 – $\frac{100}{1+RS}$

iii. Odd-lot index = $\frac{\text{Odd-lot sales}}{\text{Odd-lot purchases}}$

iv. Odd-lot short sales ratio = $\frac{\text{Odd-lot short sales}}{\text{Total odd-lot sales}}$

v. Stochastics (% K) = $\frac{C - L}{H - L}$ x 100

Where,

C = Latest closing price

L = Low price during the last N periods

H = High price during the last N periods

N = Number of periods

% D = Derived by smoothening % K using the simple moving average technique.

4. Warrants and Convertibles

i. Percentage of downside risk =

$$\frac{\left(\text{Market price of convertible security} - \text{Price of an equivalent non-convertible security}\right)}{\text{Price of an equivalent non-convertible security}} \times 100$$

ii. Conversion premium = $\frac{\text{Market price} - \text{Conversion value}}{\text{Conversion value}} \times 100$

iii. Conversion parity price = $\frac{\text{Bond price}}{\text{Number of shares on conversion per warrant}}$

iv. Break even period = $\frac{\text{Conversion premium}}{\text{Interest income} - \text{Dividends}}$

v. Payback period = $\frac{\frac{\%\ \text{premium}}{1 + \%\ \text{premium}}}{\text{Current yield} - \frac{\text{Dividend yield}}{1 + \%\ \text{premium}}}$

5. Real Assets and Mutual Funds

i. $MV_0 = \sum_{t=1}^{n} \frac{NOI_t}{(1+r)^t} + \frac{MV_n}{(1+r)^n}$

Where,

MV_0 = The current market price of the property

MV_n = The expected sales price of the property

r = The required rate of return

NOI_t = Net operating income at time t

ii. If operating income grows at the rate 'g' annually,

$MV_0 = \frac{NOI}{r - g}$

Where,

NOI = Net operating income

g = Growth rate

r = Required return

iii. Net Asset Value (NAV) = $\frac{\text{Assets} - \text{Liabilities}}{\text{No. of units outstanding}}$

XII. Strategic Financial Management

1. Capital Structure

i. Relation between EBIT and EPS

$$EPS = \frac{(EBIT - I)(1-t)}{n}$$

Where,

EBIT	=	Earning Before Interest and Tax
EPS	=	Earning per share
I	=	Interest payment
t	=	Tax rate
n	=	Number of shares

ii. EBIT – EPS Indifference Point = $\frac{(EBIT - I_1)(1-t)}{n_1} = \frac{(EBIT - I_2)(1-t)}{n_2}$

Where,

EPS	=	Earning Per Share
I_1 & I_2	=	Interest payment under alternative one and interest payment under alternative two respectively

iii. Relation between ROI and ROE

ROE = {ROI + (ROI – k_d) D/E} (I – t)

Where,

ROE	=	The Return on Equity
ROI	=	The Return on Investment
k_d	=	The cost of debt (pre-tax)
D	=	The debt component in the total capital
E	=	The equity component in the total capital
t	=	The tax rate.

2. Decision Support Models

i. Extended Probabilistic Analysis

$$C_1 = C_0 + \tilde{n}\tilde{s} - v\tilde{n}\tilde{s} - \tilde{n}f - \tilde{n}i - T(\tilde{n}\tilde{s} - v\tilde{n}\tilde{s} - \tilde{n}f - \tilde{n}i - \tilde{n}f')$$

Where,

C_1	=	Ending cash balance
C_0	=	Beginning cash balance
$\tilde{n}$	=	Duration of the recession in months
$\tilde{s}$	=	Monthly sales during the recession
$\tilde{n}\tilde{s}$	=	Total sales during the recession
v	=	Proportion of variable cash expenses to sales
$v\tilde{n}\tilde{s}$	=	Total variable cash expenses during the recession

f = Monthly fixed cash expenses, other than debt servicing burden, during the recession

ñf = Total fixed cash expenses, other than debt servicing burden during the recession

i = Monthly interest payment associated with the contemplated level of debt during the recession

ñi = Total interest payment associated with the contemplated level of debt during the recession

f′ = Monthly non-cash fixed expenses

nf′ = Total non-cash fixed expenses during the recession

T = Corporate income tax rate.

3. Working Capital Management

i. Discriminant Analysis

$Z_i = aX_i + bY_i$

Where,

Z_i = The Z-score for the ith account

X_i = The value of the first independent variable for the ith account

Y_i = The value of the second independent variable for the ith account a and b are the parameter values

$$a = \frac{\sigma_y^2.d_x - \sigma_{xy}.d_y}{\sigma_x^2.\sigma_y^2 - (\sigma_{xy})^2}$$

$$b = \frac{\sigma_x^2.d_y - \sigma_{xy}.d_x}{\sigma_x^2.\sigma_y^2 - (\sigma_{xy})^2}$$

Where,

σ_x^2 = Variance of X (across groups 1 and 2)

σ_{xy} = Covariance of X and Y (across groups 1 and 2)

σ_y^2 = Variance of Y (across groups 1 and 2)

d_x = Difference between the mean values of X for groups 1 and 2

d_y = Difference between the mean values of Y for groups 1 and 2

ii. Cash Management Models

a. Baumol Model, TC = I (C/2) + b (T/C)

Where,

TC = Total costs (total conversion costs + total holding costs)

I = Interest rate on marketable securities per planning period

C = Amount of securities liquidated per batch

T = Estimated cash requirement over the planning period

The point where total costs are minimum:

$$C = \sqrt{\frac{2bT}{I}}$$

b = Fixed conversion cost

b. Miller and Orr Model

$$RP = \sqrt[3]{\frac{3b\sigma^2}{4I}} + LL \text{ and,}$$

$$UL = 3\,RP - 2\,LL$$

Where,

LL = Lower control limit

RP = Return point

UL = Upper control limit

b = Fixed conversion cost

I = Interest rate per day on marketable securities

σ^2 = Variance of daily changes in the expected cash balance.

4. Firms in Financial Distress

i. Altman's Z-Score Model (to identify the financial distress of the firm)

$$Z = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 1.0X_5$$

Where,

Z = Discriminant score

X_1 = Working capital/Total assets

X_2 = Retained earnings/Total assets

X_3 = EBIT/Total assets

X_4 = Market value of equity/Book value of debt

X_5 = Sales/Total assets.

5. Valuation of Firms

i. a. Free cashflow of a firm = Free cashflow from operations + Non-operating cashflows

b. Free cash flows from operations = Gross cash flows of the firm – Gross investments

c. Gross cashflows of the firm = EBIT (1– T) + Depreciation + Non-cash charges

d. Gross Investment = Increase in Net Working Capital + Capital Expenditure incurred + Increase in Other Assets.

6. Mergers and Acquisitions

i. Net Acquisition Value

$$NAV = PV_{ab} - (PV_a + PV_b) - P - E$$

Where,

NAV = The Net Acquisition Value

PV_{ab} = The present value of the merged entity

PV_a = The present value of firm A

PV_b = The present value of firm B

P = The premium paid by Firm A to acquire Firm B

E = The expenses involved in the merger

ii. Conn & Nielson Model

a. Maximum Exchange Ratio acceptable to the Acquiring company

$$ER = \frac{-S_1}{S_2} + \frac{(E_1 + E_2)\,PE_{12}}{P_1\,S_2}$$

Where,

ER = Exchange Ratio

E_1 & E_2 = Earnings per Share of acquiring and target companies respectively

P_1 = Market price per share of acquiring company

PE_{12} = Price to Earnings Multiple of merged entity

S_1 & S_2 = Number of Shares outstanding in acquiring and target companies respectively

b. Minimum Exchange Ratio Acceptable to the Target Company

$$ER = \frac{P_2 S_1}{(PE_{12})(E_1 + E_2) - P_2 S_2}$$

Where,

ER = Exchange Ratio

P_2 = Market price per share of target company

E_1 & E_2 = Earnings per share of acquiring and target companies respectively

PE_{12} = Price to Earnings Multiple of merged entity

S_1 & S_2 = Number of shares outstanding in acquiring and target companies respectively.

TABLES

INTEREST RATE TABLES

Table A.1: Future Value Interest Factor

$FV = PV(1 + k)^n$

n/i	1.0%	2.0%	3.0%	4.0%	5.0%	6.0%	7.0%	8.0%	9.0%	10.0%
1	1.0100	1.0200	1.0300	1.0400	1.0500	1.0600	1.0700	1.0800	1.0900	1.1000
2	1.0201	1.0404	1.0609	1.0816	1.1025	1.1236	1.1449	1.1664	1.1881	1.2100
3	1.0303	1.0612	1.0927	1.1249	1.1576	1.1910	1.2250	1.2597	1.2950	1.3310
4	1.0406	1.0824	1.1255	1.1699	1.2155	1.2625	1.3108	1.3605	1.4116	1.4641
5	1.0510	1.1041	1.1593	1.2167	1.2763	1.3382	1.4026	1.4693	1.5386	1.6105
6	1.0615	1.1262	1.1941	1.2653	1.3401	1.4185	1.5007	1.5869	1.6771	1.7716
7	1.0721	1.1487	1.2299	1.3159	1.4071	1.5036	1.6058	1.7138	1.8280	1.9487
8	1.0829	1.1717	1.2668	1.3686	1.4775	1.5938	1.7182	1.8509	1.9926	2.1436
9	1.0937	1.1951	1.3048	1.4233	1.5513	1.6895	1.8385	1.9990	2.1719	2.3579
10	1.1046	1.2190	1.3439	1.4802	1.6289	1.7908	1.9672	2.1589	2.3674	2.5937
11	1.1157	1.2434	1.3842	1.5395	1.7103	1.8983	2.1049	2.3316	2.5804	2.8531
12	1.1268	1.2682	1.4258	1.6010	1.7959	2.0122	2.2522	2.5182	2.8127	3.1384
13	1.1381	1.2936	1.4685	1.6651	1.8856	2.1329	2.4098	2.7196	3.0658	3.4523
14	1.1495	1.3195	1.5126	1.7317	1.9799	2.2609	2.5785	2.9372	3.3417	3.7975
15	1.1610	1.3459	1.5580	1.8009	2.0789	2.3966	2.7590	3.1722	3.6425	4.1772
16	1.1726	1.3728	1.6047	1.8730	2.1829	2.5404	2.9522	3.4259	3.9703	4.5950
17	1.1843	1.4002	1.6528	1.9479	2.2920	2.6928	3.1588	3.7000	4.3276	5.0545
18	1.1961	1.4282	1.7024	2.0258	2.4066	2.8543	3.3799	3.9960	4.7171	5.5599
19	1.2081	1.4568	1.7535	2.1068	2.5270	3.0256	3.6165	4.3157	5.1417	6.1159
20	1.2202	1.4859	1.8061	2.1911	2.6533	3.2071	3.8697	4.6610	5.6044	6.7275
21	1.2324	1.5157	1.8603	2.2788	2.7860	3.3996	4.1406	5.0338	6.1088	7.4002
22	1.2447	1.5460	1.9161	2.3699	2.9253	3.6035	4.4304	5.4365	6.6586	8.1403
23	1.2572	1.5769	1.9736	2.4647	3.0715	3.8197	4.7405	5.8715	7.2579	8.9543
24	1.2697	1.6084	2.0328	2.5633	3.2251	4.0489	5.0724	6.3412	7.9111	9.8497
25	1.2824	1.6406	2.0938	2.6658	3.3864	4.2919	5.4274	6.8485	8.6231	10.8347
26	1.2953	1.6734	2.1566	2.7725	3.5557	4.5494	5.8074	7.3964	9.3992	11.9182
27	1.3082	1.7069	2.2213	2.8834	3.7335	4.8223	6.2139	7.9881	10.2451	13.1100
28	1.3213	1.7410	2.2879	2.9987	3.9201	5.1117	6.6488	8.6271	11.1671	14.4210
29	1.3345	1.7758	2.3566	3.1187	4.1161	5.4184	7.1143	9.3173	12.1722	15.8631
30	1.3478	1.8114	2.4273	3.2434	4.3219	5.7435	7.6123	10.0627	13.2677	17.4494
40	1.4889	2.2080	3.2620	4.8010	7.0400	10.2857	14.9745	21.7245	31.4094	45.2593
50	1.6446	2.6916	4.3839	7.1067	11.4674	18.4202	29.4570	46.9016	74.3575	117.3909
60	1.8167	3.2810	5.8916	10.5196	18.6792	32.9877	57.9464	101.2571	176.0313	304.4816

n/i	12.0%	14.0%	15.0%	16.0%	18.0%	20.0%	24.0%	28.0%	32.0%	36.0%
1	1.1200	1.1400	1.1500	1.1600	1.1800	1.2000	1.2400	1.2800	1.3200	1.3600
2	1.2544	1.2996	1.3225	1.3456	1.3924	1.4400	1.5376	1.6384	1.7424	1.8496
3	1.4049	1.4815	1.5209	1.5609	1.6430	1.7280	1.9066	2.0972	2.3000	2.5155
4	1.5735	1.6890	1.7490	1.8106	1.9388	2.0736	2.3642	2.6844	3.0360	3.4210
5	1.7623	1.9254	2.0114	2.1003	2.2878	2.4883	2.9316	3.4360	4.0075	4.6526
6	1.9738	2.1950	2.3131	2.4364	2.6996	2.9860	3.6352	4.3980	5.2899	6.3275
7	2.2107	2.5023	2.6600	2.8262	3.1855	3.5832	4.5077	5.6295	6.9826	8.6054
8	2.4760	2.8526	3.0590	3.2784	3.7589	4.2998	5.5895	7.2058	9.2170	11.7034
9	2.7731	3.2519	3.5179	3.8030	4.4355	5.1598	6.9310	9.2234	12.1665	15.9166
10	3.1058	3.7072	4.0456	4.4114	5.2338	6.1917	8.5944	11.8059	16.0598	21.6466
11	3.4785	4.2262	4.6524	5.1173	6.1759	7.4301	10.6571	15.1116	21.1989	29.4393
12	3.8960	4.8179	5.3503	5.9360	7.2876	8.9161	13.2148	19.3428	27.9825	40.0375
13	4.3635	5.4924	6.1528	6.8858	8.5994	10.6993	16.3863	24.7588	36.9370	54.4510
14	4.8871	6.2613	7.0757	7.9875	10.1472	12.8392	20.3191	31.6913	48.7568	74.0534
15	5.4736	7.1379	8.1371	9.2655	11.9737	15.4070	25.1956	40.5648	64.3590	100.7126
16	6.1304	8.1372	9.3576	10.7480	14.1290	18.4884	31.2426	51.9230	84.9538	136.9691
17	6.8660	9.2765	10.7613	12.4677	16.6722	22.1861	38.7408	66.4614	112.1390	186.2779
18	7.6900	10.5752	12.3755	14.4625	19.6733	26.6233	48.0386	85.0706	148.0235	253.3380
19	8.6128	12.0557	14.2318	16.7765	23.2144	31.9480	59.5679	108.8904	195.3911	344.5397
20	9.6463	13.7435	16.3665	19.4608	27.3930	38.3376	73.8641	139.3797	257.9162	468.5740
21	10.8038	15.6676	18.8215	22.5745	32.3238	46.0051	91.5915	178.4060	340.4494	637.2606
22	12.1003	17.8610	21.6447	26.1864	38.1421	55.2061	113.5735	228.3596	449.3932	866.6744
23	13.5523	20.3616	24.8915	30.3762	45.0076	66.2474	140.8312	292.3003	593.1990	1178.6772
24	15.1786	23.2122	28.6252	35.2364	53.1090	79.4968	174.6306	374.1444	783.0227	1603.0010
25	17.0001	26.4619	32.9190	40.8742	62.6686	95.3962	216.5420	478.9049	1033.5900	2180.0814
26	19.0401	30.1666	37.8568	47.4141	73.9490	114.4755	268.5121	612.9982	1364.3387	2964.9107
27	21.3249	34.3899	43.5353	55.0004	87.2598	137.3706	332.9550	784.6377	1800.9271	4032.2786
28	23.8839	39.2045	50.0656	63.8004	102.9666	164.8447	412.8642	1004.3363	2377.2238	5483.8988
29	26.7499	44.6931	57.5755	74.0085	121.5005	197.8136	511.9516	1285.5504	3137.9354	7458.1024
30	29.9599	50.9502	66.2118	85.8499	143.3706	237.3763	634.8199	1645.5046	4142.0748	10143.0193
40	93.0510	188.8835	267.8635	378.7212	750.3783	1469.7716	5455.9126	19426.6889	66520.7670	219561.5736
50	289.0022	700.2330	1083.6574	1670.7038	3927.3569	9100.4382	46890.4346	229349.8616	1068308.1960	4752754.9027
60	897.5969	2595.9187	4383.9987	7370.2014	20555.1400	56347.5144	402996.3473	2707685.2482	17156783.5543	102880840.1651

Table A.2 : Future Value Interest Factor for an Annuity

$$FVIFA(k, n) = \frac{(1+k)^n - 1}{k}$$

n/i	1.0%	2.0%	3.0%	4.0%	5.0%	6.0%	7.0%	8.0%	9.0%	10.0%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500	2.0600	2.0700	2.0800	2.0900	2.1000
3	3.0301	3.0604	3.0909	3.1216	3.1525	3.1836	3.2149	3.2464	3.2781	3.3100
4	4.0604	4.1216	4.1836	4.2465	4.3101	4.3746	4.4399	4.5061	4.5731	4.6410
5	5.1010	5.2040	5.3091	5.4163	5.5256	5.6371	5.7507	5.8666	5.9847	6.1051
6	6.1520	6.3081	6.4684	6.6330	6.8019	6.9753	7.1533	7.3359	7.5233	7.7156
7	7.2135	7.4343	7.6625	7.8983	8.1420	8.3938	8.6540	8.9228	9.2004	9.4872
8	8.2857	8.5830	8.8923	9.2142	9.5491	9.8975	10.2598	10.6366	11.0285	11.4359
9	9.3685	9.7546	10.1591	10.5828	11.0266	11.4913	11.9780	12.4876	13.0210	13.5795
10	10.4622	10.9497	11.4639	12.0061	12.5779	13.1808	13.8164	14.4866	15.1929	15.9374
11	11.5668	12.1687	12.8078	13.4864	14.2068	14.9716	15.7836	16.6455	17.5603	18.5312
12	12.6825	13.4121	14.1920	15.0258	15.9171	16.8699	17.8885	18.9771	20.1407	21.3843
13	13.8093	14.6803	15.6178	16.6268	17.7130	18.8821	20.1406	21.4953	22.9534	24.5227
14	14.9474	15.9739	17.0863	18.2919	19.5986	21.0151	22.5505	24.2149	26.0192	27.9750
15	16.0969	17.2934	18.5989	20.0236	21.5786	23.2760	25.1290	27.1521	29.3609	31.7725
16	17.2579	18.6393	20.1569	21.8245	23.6575	25.6725	27.8881	30.3243	33.0034	35.9497
17	18.4304	20.0121	21.7616	23.6975	25.8404	28.2129	30.8402	33.7502	36.9737	40.5447
18	19.6147	21.4123	23.4144	25.6454	28.1324	30.9057	33.9990	37.4502	41.3013	45.5992
19	20.8109	22.8406	25.1169	27.6712	30.5390	33.7600	37.3790	41.4463	46.0185	51.1591
20	22.0190	24.2974	26.8704	29.7781	33.0660	36.7856	40.9955	45.7620	51.1601	57.2750
21	23.2392	25.7833	28.6765	31.9692	35.7193	39.9927	44.8652	50.4229	56.7645	64.0025
22	24.4716	27.2990	30.5368	34.2480	38.5052	43.3923	49.0057	55.4568	62.8733	71.4027
23	25.7163	28.8450	32.4529	36.6179	41.4305	46.9958	53.4361	60.8933	69.5319	79.5430
24	26.9735	30.4219	34.4265	39.0826	44.5020	50.8156	58.1767	66.7648	76.7898	88.4973
25	28.2432	32.0303	36.4593	41.6459	47.7271	54.8645	63.2490	73.1059	84.7009	98.3471
26	29.5256	33.6709	38.5530	44.3117	51.1135	59.1564	68.6765	79.9544	93.3240	109.1818
27	30.8209	35.3443	40.7096	47.0842	54.6691	63.7058	74.4838	87.3508	102.7231	121.0999
28	32.1291	37.0512	42.9309	49.9676	58.4026	68.5281	80.6977	95.3388	112.9682	134.2099
29	33.4504	38.7922	45.2189	52.9663	62.3227	73.6398	87.3465	103.9659	124.1354	148.6309
30	34.7849	40.5681	47.5754	56.0849	66.4388	79.0582	94.4608	113.2832	136.3075	164.4940
40	48.8864	60.4020	75.4013	95.0255	120.7998	154.7620	199.6351	259.0565	337.8824	442.5926
50	64.4632	84.5794	112.7969	152.6671	209.3480	290.3359	406.5289	573.7702	815.0836	1163.9085
60	81.6697	114.0515	163.0534	237.9907	353.5837	533.1282	813.5204	1253.2133	1944.7921	3034.8164

n/i	12.0%	14.0%	15.0%	16.0%	18.0%	20.0%	24.0%	28.0%	32.0%	36.0%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.1200	2.1400	2.1500	2.1600	2.1800	2.2000	2.2400	2.2800	2.3200	2.3600
3	3.3744	3.4396	3.4725	3.5056	3.5724	3.6400	3.7776	3.9184	4.0624	4.2096
4	4.7793	4.9211	4.9934	5.0665	5.2154	5.3680	5.6842	6.0156	6.3624	6.7251
5	6.3528	6.6101	6.7424	6.8771	7.1542	7.4416	8.0484	8.6999	9.3983	10.1461
6	8.1152	8.5355	8.7537	8.9775	9.4420	9.9299	10.9801	12.1359	13.4058	14.7987
7	10.0890	10.7305	11.0668	11.4139	12.1415	12.9159	14.6153	16.5339	18.6956	21.1262
8	12.2997	13.2328	13.7268	14.2401	15.3270	16.4991	19.1229	22.1634	25.6782	29.7316
9	14.7757	16.0853	16.7858	17.5185	19.0859	20.7989	24.7125	29.3692	34.8953	41.4350
10	17.5487	19.3373	20.3037	21.3215	23.5213	25.9587	31.6434	38.5926	47.0618	57.3516
11	20.6546	23.0445	24.3493	25.7329	28.7551	32.1504	40.2379	50.3985	63.1215	78.9982
12	24.1331	27.2707	29.0017	30.8502	34.9311	39.5805	50.8950	65.5100	84.3204	108.4375
13	28.0291	32.0887	34.3519	36.7862	42.2187	48.4966	64.1097	84.8529	112.3030	148.4750
14	32.3926	37.5811	40.5047	43.6720	50.8180	59.1959	80.4961	109.6117	149.2399	202.9260
15	37.2797	43.8424	47.5804	51.6595	60.9653	72.0351	100.8151	141.3029	197.9967	276.9793
16	42.7533	50.9804	55.7175	60.9250	72.9390	87.4421	126.0108	181.8677	262.3557	377.6919
17	48.8837	59.1176	65.0751	71.6730	87.0680	105.9306	157.2534	233.7907	347.3095	514.6610
18	55.7497	68.3941	75.8364	84.1407	103.7403	128.1167	195.9942	300.2521	459.4485	700.9389
19	63.4397	78.9692	88.2118	98.6032	123.4135	154.7400	244.0328	385.3227	607.4721	954.2769
20	72.0524	91.0249	102.4436	115.3797	146.6280	186.6880	303.6006	494.2131	802.8631	1298.8166
21	81.6987	104.7684	118.8101	134.8405	174.0210	225.0256	377.4648	633.5927	1060.7793	1767.3906
22	92.5026	120.4360	137.6316	157.4150	206.3448	271.0307	469.0563	811.9987	1401.2287	2404.6512
23	104.6029	138.2970	159.2764	183.6014	244.4868	326.2369	582.6298	1040.3583	1850.6219	3271.3256
24	118.1552	158.6586	184.1678	213.9776	289.4945	392.4842	723.4610	1332.6586	2443.8209	4450.0029
25	133.3339	181.8708	212.7930	249.2140	342.6035	471.9811	898.0916	1706.8031	3226.8436	6053.0039
26	150.3339	208.3327	245.7120	290.0883	405.2721	567.3773	1114.6336	2185.7079	4260.4336	8233.0853
27	169.3740	238.4993	283.5688	337.5023	479.2211	681.8528	1383.1457	2798.7061	5624.7723	11197.9960
28	190.6989	272.8892	327.1041	392.5028	566.4809	819.2233	1716.1007	3583.3438	7425.6994	15230.2745
29	214.5828	312.0937	377.1697	456.3032	669.4475	984.0680	2128.9648	4587.6801	9802.9233	20714.1734
30	241.3327	356.7868	434.7451	530.3117	790.9480	1181.8816	2640.9164	5873.2306	12940.8587	28172.2758
40	767.0914	1342.0251	1779.0903	2360.7572	4163.2130	7343.8578	22728.8026	69377.4604	207874.2719	609890.4824
50	2400.0182	4994.5213	7217.7163	10435.6488	21813.0937	45497.1908	195372.6442	819103.0771	3338459.9875	13202094.1741
60	7471.6411	18535.1333	29219.9916	46057.5085	114189.6665	281732.5718	1679147.2802	9670300.8863	53614945.4823	285780108.7920

Table A.3 : Present Value Interest Factor

$$PV = \frac{1}{(1+k)^n}$$

n/i	1.0%	2.0%	3.0%	4.0%	5.0%	6.0%	7.0%	8.0%	9.0%	10.0%
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9346	0.9259	0.9174	0.9091
2	0.9803	0.9612	0.9426	0.9246	0.9070	0.8900	0.8734	0.8573	0.8417	0.8264
3	0.9706	0.9423	0.9151	0.8890	0.8638	0.8396	0.8163	0.7938	0.7722	0.7513
4	0.9610	0.9238	0.8885	0.8548	0.8227	0.7921	0.7629	0.7350	0.7084	0.6830
5	0.9515	0.9057	0.8626	0.8219	0.7835	0.7473	0.7130	0.6806	0.6499	0.6209
6	0.9420	0.8880	0.8375	0.7903	0.7462	0.7050	0.6663	0.6302	0.5963	0.5645
7	0.9327	0.8706	0.8131	0.7599	0.7107	0.6651	0.6227	0.5835	0.5470	0.5132
8	0.9235	0.8535	0.7894	0.7307	0.6768	0.6274	0.5820	0.5403	0.5019	0.4665
9	0.9143	0.8368	0.7664	0.7026	0.6446	0.5919	0.5439	0.5002	0.4604	0.4241
10	0.9053	0.8203	0.7441	0.6756	0.6139	0.5584	0.5083	0.4632	0.4224	0.3855
11	0.8963	0.8043	0.7224	0.6496	0.5847	0.5268	0.4751	0.4289	0.3875	0.3505
12	0.8874	0.7885	0.7014	0.6246	0.5568	0.4970	0.4440	0.3971	0.3555	0.3186
13	0.8787	0.7730	0.6810	0.6006	0.5303	0.4688	0.4150	0.3677	0.3262	0.2897
14	0.8700	0.7579	0.6611	0.5775	0.5051	0.4423	0.3878	0.3405	0.2992	0.2633
15	0.8613	0.7430	0.6419	0.5553	0.4810	0.4173	0.3624	0.3152	0.2745	0.2394
16	0.8528	0.7284	0.6232	0.5339	0.4581	0.3936	0.3387	0.2919	0.2519	0.2176
17	0.8444	0.7142	0.6050	0.5134	0.4363	0.3714	0.3166	0.2703	0.2311	0.1978
18	0.8360	0.7002	0.5874	0.4936	0.4155	0.3503	0.2959	0.2502	0.2120	0.1799
19	0.8277	0.6864	0.5703	0.4746	0.3957	0.3305	0.2765	0.2317	0.1945	0.1635
20	0.8195	0.6730	0.5537	0.4564	0.3769	0.3118	0.2584	0.2145	0.1784	0.1486
21	0.8114	0.6598	0.5375	0.4388	0.3589	0.2942	0.2415	0.1987	0.1637	0.1351
22	0.8034	0.6468	0.5219	0.4220	0.3418	0.2775	0.2257	0.1839	0.1502	0.1228
23	0.7954	0.6342	0.5067	0.4057	0.3256	0.2618	0.2109	0.1703	0.1378	0.1117
24	0.7876	0.6217	0.4919	0.3901	0.3101	0.2470	0.1971	0.1577	0.1264	0.1015
25	0.7798	0.6095	0.4776	0.3751	0.2953	0.2330	0.1842	0.1460	0.1160	0.0923
26	0.7720	0.5976	0.4637	0.3607	0.2812	0.2198	0.1722	0.1352	0.1064	0.0839
27	0.7644	0.5859	0.4502	0.3468	0.2678	0.2074	0.1609	0.1252	0.0976	0.0763
28	0.7568	0.5744	0.4371	0.3335	0.2551	0.1956	0.1504	0.1159	0.0895	0.0693
29	0.7493	0.5631	0.4243	0.3207	0.2429	0.1846	0.1406	0.1073	0.0822	0.0630
30	0.7419	0.5521	0.4120	0.3083	0.2314	0.1741	0.1314	0.0994	0.0754	0.0573
40	0.6717	0.4529	0.3066	0.2083	0.1420	0.0972	0.0668	0.0460	0.0318	0.0221
50	0.6080	0.3715	0.2281	0.1407	0.0872	0.0543	0.0339	0.0213	0.0134	0.0085
60	0.5504	0.3048	0.1697	0.0951	0.0535	0.0303	0.0173	0.0099	0.0057	0.0033

n/i	12.0%	14.0%	15.0%	16.0%	18.0%	20.0%	24.0%	28.0%	32.0%	36.0%
1	0.8929	0.8772	0.8696	0.8621	0.8475	0.8333	0.8065	0.7813	0.7576	0.7353
2	0.7972	0.7695	0.7561	0.7432	0.7182	0.6944	0.6504	0.6104	0.5739	0.5407
3	0.7118	0.6750	0.6575	0.6407	0.6086	0.5787	0.5245	0.4768	0.4348	0.3975
4	0.6355	0.5921	0.5718	0.5523	0.5158	0.4823	0.4230	0.3725	0.3294	0.2923
5	0.5674	0.5194	0.4972	0.4761	0.4371	0.4019	0.3411	0.2910	0.2495	0.2149
6	0.5066	0.4556	0.4323	0.4104	0.3704	0.3349	0.2751	0.2274	0.1890	0.1580
7	0.4523	0.3996	0.3759	0.3538	0.3139	0.2791	0.2218	0.1776	0.1432	0.1162
8	0.4039	0.3506	0.3269	0.3050	0.2660	0.2326	0.1789	0.1388	0.1085	0.0854
9	0.3606	0.3075	0.2843	0.2630	0.2255	0.1938	0.1443	0.1084	0.0822	0.0628
10	0.3220	0.2697	0.2472	0.2267	0.1911	0.1615	0.1164	0.0847	0.0623	0.0462
11	0.2875	0.2366	0.2149	0.1954	0.1619	0.1346	0.0938	0.0662	0.0472	0.0340
12	0.2567	0.2076	0.1869	0.1685	0.1372	0.1122	0.0757	0.0517	0.0357	0.0250
13	0.2292	0.1821	0.1625	0.1452	0.1163	0.0935	0.0610	0.0404	0.0271	0.0184
14	0.2046	0.1597	0.1413	0.1252	0.0985	0.0779	0.0492	0.0316	0.0205	0.0135
15	0.1827	0.1401	0.1229	0.1079	0.0835	0.0649	0.0397	0.0247	0.0155	0.0099
16	0.1631	0.1229	0.1069	0.0930	0.0708	0.0541	0.0320	0.0193	0.0118	0.0073
17	0.1456	0.1078	0.0929	0.0802	0.0600	0.0451	0.0258	0.0150	0.0089	0.0054
18	0.1300	0.0946	0.0808	0.0691	0.0508	0.0376	0.0208	0.0118	0.0068	0.0039
19	0.1161	0.0829	0.0703	0.0596	0.0431	0.0313	0.0168	0.0092	0.0051	0.0029
20	0.1037	0.0728	0.0611	0.0514	0.0365	0.0261	0.0135	0.0072	0.0039	0.0021
21	0.0926	0.0638	0.0531	0.0443	0.0309	0.0217	0.0109	0.0056	0.0029	0.0016
22	0.0826	0.0560	0.0462	0.0382	0.0262	0.0181	0.0088	0.0044	0.0022	0.0012
23	0.0738	0.0491	0.0402	0.0329	0.0222	0.0151	0.0071	0.0034	0.0017	0.0008
24	0.0659	0.0431	0.0349	0.0284	0.0188	0.0126	0.0057	0.0027	0.0013	0.0006
25	0.0588	0.0378	0.0304	0.0245	0.0160	0.0105	0.0046	0.0021	0.0010	0.0005
26	0.0525	0.0331	0.0264	0.0211	0.0135	0.0087	0.0037	0.0016	0.0007	0.0003
27	0.0469	0.0291	0.0230	0.0182	0.0115	0.0073	0.0030	0.0013	0.0006	0.0002
28	0.0419	0.0255	0.0200	0.0157	0.0097	0.0061	0.0024	0.0010	0.0004	0.0002
29	0.0374	0.0224	0.0174	0.0135	0.0082	0.0051	0.0020	0.0008	0.0003	0.0001
30	0.0334	0.0196	0.0151	0.0116	0.0070	0.0042	0.0016	0.0006	0.0002	0.0001
40	0.0107	0.0053	0.0037	0.0026	0.0013	0.0007	0.0002	0.0001	0.0000	0.0000
50	0.0035	0.0014	0.0009	0.0006	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000
60	0.0011	0.0004	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

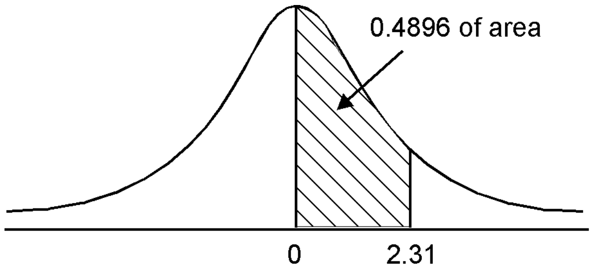
Table A.4 : Present Value Interest Factor for an Annuity

$$PVIFA(k, n) = \frac{(1+k)^n - 1}{k(1+k)^n}$$

n/i	1.0%	2.0%	3.0%	4.0%	5.0%	6.0%	7.0%	8.0%	9.0%	10.0%
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9346	0.9259	0.9174	0.9091
2	1.9704	1.9416	1.9135	1.8861	1.8594	1.8334	1.8080	1.7833	1.7591	1.7355
3	2.9410	2.8839	2.8286	2.7751	2.7232	2.6730	2.6243	2.5771	2.5313	2.4869
4	3.9020	3.8077	3.7171	3.6299	3.5460	3.4651	3.3872	3.3121	3.2397	3.1699
5	4.8534	4.7135	4.5797	4.4518	4.3295	4.2124	4.1002	3.9927	3.8897	3.7908
6	5.7955	5.6014	5.4172	5.2421	5.0757	4.9173	4.7665	4.6229	4.4859	4.3553
7	6.7282	6.4720	6.2303	6.0021	5.7864	5.5824	5.3893	5.2064	5.0330	4.8684
8	7.6517	7.3255	7.0197	6.7327	6.4632	6.2098	5.9713	5.7466	5.5348	5.3349
9	8.5660	8.1622	7.7861	7.4353	7.1078	6.8017	6.5152	6.2469	5.9952	5.7590
10	9.4713	8.9826	8.5302	8.1109	7.7217	7.3601	7.0236	6.7101	6.4177	6.1446
11	10.3676	9.7868	9.2526	8.7605	8.3064	7.8869	7.4987	7.1390	6.8052	6.4951
12	11.2551	10.5753	9.9540	9.3851	8.8633	8.3838	7.9427	7.5361	7.1607	6.8137
13	12.1337	11.3484	10.6350	9.9856	9.3936	8.8527	8.3577	7.9038	7.4869	7.1034
14	13.0037	12.1062	11.2961	10.5631	9.8986	9.2950	8.7455	8.2442	7.7862	7.3667
15	13.8651	12.8493	11.9379	11.1184	10.3797	9.7122	9.1079	8.5595	8.0607	7.6061
16	14.7179	13.5777	12.5611	11.6523	10.8378	10.1059	9.4466	8.8514	8.3126	7.8237
17	15.5623	14.2919	13.1661	12.1657	11.2741	10.4773	9.7632	9.1216	8.5436	8.0216
18	16.3983	14.9920	13.7535	12.6593	11.6896	10.8276	10.0591	9.3719	8.7556	8.2014
19	17.2260	15.6785	14.3238	13.1339	12.0853	11.1581	10.3356	9.6036	8.9501	8.3649
20	18.0456	16.3514	14.8775	13.5903	12.4622	11.4699	10.5940	9.8181	9.1285	8.5136
21	18.8570	17.0112	15.4150	14.0292	12.8212	11.7641	10.8355	10.0168	9.2922	8.6487
22	19.6604	17.6580	15.9369	14.4511	13.1630	12.0416	11.0612	10.2007	9.4424	8.7715
23	20.4558	18.2922	16.4436	14.8568	13.4886	12.3034	11.2722	10.3711	9.5802	8.8832
24	21.2434	18.9139	16.9355	15.2470	13.7986	12.5504	11.4693	10.5288	9.7066	8.9847
25	22.0232	19.5235	17.4131	15.6221	14.0939	12.7834	11.6536	10.6748	9.8226	9.0770
26	22.7952	20.1210	17.8768	15.9828	14.3752	13.0032	11.8258	10.8100	9.9290	9.1609
27	23.5596	20.7069	18.3270	16.3296	14.6430	13.2105	11.9867	10.9352	10.0266	9.2372
28	24.3164	21.2813	18.7641	16.6631	14.8981	13.4062	12.1371	11.0511	10.1161	9.3066
29	25.0658	21.8444	19.1885	16.9837	15.1411	13.5907	12.2777	11.1584	10.1983	9.3696
30	25.8077	22.3965	19.6004	17.2920	15.3725	13.7648	12.4090	11.2578	10.2737	9.4269
40	32.8347	27.3555	23.1148	19.7928	17.1591	15.0463	13.3317	11.9246	10.7574	9.7791
50	39.1961	31.4236	25.7298	21.4822	18.2559	15.7619	13.8007	12.2335	10.9617	9.9148
60	44.9550	34.7609	27.6756	22.6235	18.9293	16.1614	14.0392	12.3766	11.0480	9.9672

n/i	12.0%	14.0%	15.0%	16.0%	18.0%	20.0%	24.0%	28.0%	32.0%	36.0%
1	0.8929	0.8772	0.8696	0.8621	0.8475	0.8333	0.8065	0.7813	0.7576	0.7353
2	1.6901	1.6467	1.6257	1.6052	1.5656	1.5278	1.4568	1.3916	1.3315	1.2760
3	2.4018	2.3216	2.2832	2.2459	2.1743	2.1065	1.9813	1.8684	1.7663	1.6735
4	3.0373	2.9137	2.8550	2.7982	2.6901	2.5887	2.4043	2.2410	2.0957	1.9658
5	3.6048	3.4331	3.3522	3.2743	3.1272	2.9906	2.7454	2.5320	2.3452	2.1807
6	4.1114	3.8887	3.7845	3.6847	3.4976	3.3255	3.0205	2.7594	2.5342	2.3388
7	4.5638	4.2883	4.1604	4.0386	3.8115	3.6046	3.2423	2.9370	2.6775	2.4550
8	4.9676	4.6389	4.4873	4.3436	4.0776	3.8372	3.4212	3.0758	2.7860	2.5404
9	5.3282	4.9464	4.7716	4.6065	4.3030	4.0310	3.5655	3.1842	2.8681	2.6033
10	5.6502	5.2161	5.0188	4.8332	4.4941	4.1925	3.6819	3.2689	2.9304	2.6495
11	5.9377	5.4527	5.2337	5.0286	4.6560	4.3271	3.7757	3.3351	2.9776	2.6834
12	6.1944	5.6603	5.4206	5.1971	4.7932	4.4392	3.8514	3.3868	3.0133	2.7084
13	6.4235	5.8424	5.5831	5.3423	4.9095	4.5327	3.9124	3.4272	3.0404	2.7268
14	6.6282	6.0021	5.7245	5.4675	5.0081	4.6106	3.9616	3.4587	3.0609	2.7403
15	6.8109	6.1422	5.8474	5.5755	5.0916	4.6755	4.0013	3.4834	3.0764	2.7502
16	6.9740	6.2651	5.9542	5.6685	5.1624	4.7296	4.0333	3.5026	3.0882	2.7575
17	7.1196	6.3729	6.0472	5.7487	5.2223	4.7746	4.0591	3.5177	3.0971	2.7629
18	7.2497	6.4674	6.1280	5.8178	5.2732	4.8122	4.0799	3.5294	3.1039	2.7668
19	7.3658	6.5504	6.1982	5.8775	5.3162	4.8435	4.0967	3.5386	3.1090	2.7697
20	7.4694	6.6231	6.2593	5.9288	5.3527	4.8696	4.1103	3.5458	3.1129	2.7718
21	7.5620	6.6870	6.3125	5.9731	5.3837	4.8913	4.1212	3.5514	3.1158	2.7734
22	7.6446	6.7429	6.3587	6.0113	5.4099	4.9094	4.1300	3.5558	3.1180	2.7746
23	7.7184	6.7921	6.3988	6.0442	5.4321	4.9245	4.1371	3.5592	3.1197	2.7754
24	7.7843	6.8351	6.4338	6.0726	5.4509	4.9371	4.1428	3.5619	3.1210	2.7760
25	7.8431	6.8729	6.4641	6.0971	5.4669	4.9476	4.1474	3.5640	3.1220	2.7765
26	7.8957	6.9061	6.4906	6.1182	5.4804	4.9563	4.1511	3.5656	3.1227	2.7768
27	7.9426	6.9352	6.5135	6.1364	5.4919	4.9636	4.1542	3.5669	3.1233	2.7771
28	7.9844	6.9607	6.5335	6.1520	5.5016	4.9697	4.1566	3.5679	3.1237	2.7773
29	8.0218	6.9830	6.5509	6.1656	5.5098	4.9747	4.1585	3.5687	3.1240	2.7774
30	8.0552	7.0027	6.5660	6.1772	5.5168	4.9789	4.1601	3.5693	3.1242	2.7775
40	8.2438	7.1050	6.6418	6.2335	5.5482	4.9966	4.1659	3.5712	3.1250	2.7778
50	8.3045	7.1327	6.6605	6.2463	5.5541	4.9995	4.1666	3.5714	3.1250	2.7778
60	8.3240	7.1401	6.6651	6.2492	5.5553	4.9999	4.1667	3.5714	3.1250	2.7778

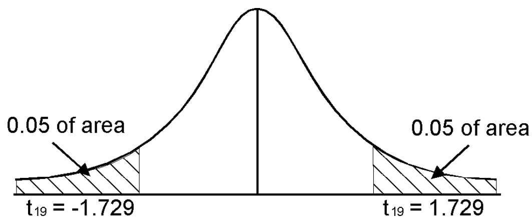
STANDARD NORMAL PROBABILITY DISTRIBUTION TABLE



$$Z = \frac{x - \mu}{\sigma}$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

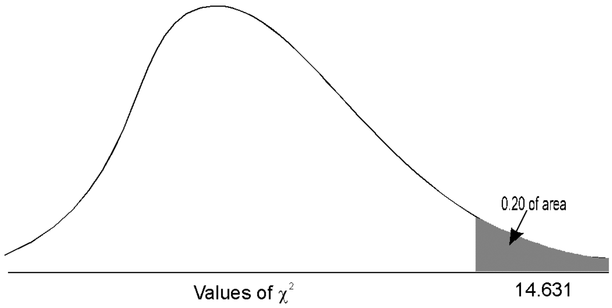
t DISTRIBUTION TABLE



$$t = \frac{x - \mu}{\sigma}$$

Degrees of freedom	Area in both tails combined			
	0.10	0.05	.02	.01
1	6.314	12.706	31.821	63.657
2	2.920	4.303	6.965	9.925
3	2.353	3.182	4.541	5.841
4	2.132	2.776	3.747	4.604
5	2.015	2.571	3.365	4.032
6	1.943	2.447	3.143	3.707
7	1.895	2.365	2.998	3.499
8	1.860	2.306	2.896	3.355
9	1.833	2.262	2.821	3.250
10	1.812	2.228	2.764	3.169
11	1.796	2.201	2.718	3.106
12	1.782	2.179	2.681	3.055
13	1.771	2.160	2.650	3.012
14	1.761	2.145	2.624	2.977
15	1.753	2.131	2.602	2.947
16	1.746	2.120	2.583	2.921
17	1.740	2.110	2.567	2.898
18	1.734	2.101	2.552	2.878
19	1.729	2.093	2.539	2.861
20	1.725	2.086	2.528	2.845
21	1.721	2.080	2.518	2.831
22	1.717	2.074	2.508	2.819
23	1.714	2.069	2.500	2.807
24	1.711	2.064	2.492	2.797
25	1.708	2.060	2.485	2.787
26	1.706	2.056	2.479	2.779
27	1.703	2.052	2.473	2.771
28	1.701	2.048	2.467	2.763
29	1.699	2.045	2.462	2.756
30	1.697	2.042	2.457	2.750
40	1.684	2.021	2.423	2.704
60	1.671	2.000	2.390	2.660
120	1.658	1.980	2.358	2.617
Normal Distribution	1.645	1.960	2.326	2.576

AREA IN THE RIGHT TAIL OF A CHI-SQUARE (χ^2) DISTRIBUTION TABLE



Degrees of freedom	Area in right tail				
	0.99	0.975	0.95	0.90	0.80
1	0.00016	0.00098	0.00398	0.0158	0.0642
2	0.0201	0.0506	0.103	0.211	0.446
3	0.115	0.216	0.352	0.584	1.005
4	0.297	0.484	0.711	1.064	1.649
5	0.554	0.831	1.145	1.610	2.343
6	0.872	1.237	1.635	2.204	3.070
7	1.239	1.690	2.167	2.833	3.822
8	1.646	2.180	2.733	3.490	4.594
9	2.088	2.700	3.325	4.168	5.380
10	2.558	3.247	3.940	4.865	6.179
11	3.053	3.816	4.575	5.578	6.989
12	3.571	4.404	5.226	6.304	7.807
13	4.107	5.009	5.892	7.042	8.634
14	4.660	5.629	6.571	7.790	9.467
15	5.229	6.262	7.261	8.547	10.307
16	5.812	6.908	7.962	9.312	11.152
17	6.408	7.564	8.672	10.085	12.002
18	7.015	8.231	9.390	10.865	12.857
19	7.633	8.907	10.117	11.651	13.716
20	8.260	9.591	10.851	12.443	14.578
21	8.897	10.283	11.591	13.240	15.445
22	9.542	10.982	12.338	14.041	16.314
23	10.196	11.689	13.091	14.848	17.187
24	10.856	12.401	13.848	15.658	18.062
25	11.524	13.120	14.611	16.473	18.940
26	12.198	13.844	15.379	17.292	19.820
27	12.879	14.573	16.151	18.114	20.703
28	13.565	15.308	16.928	18.939	21.588
29	14.256	16.047	17.708	19.768	22.475
30	14.953	16.791	18.493	20.599	23.364

AREA IN THE RIGHT TAIL OF A CHI-SQUARE (χ^2) DISTRIBUTION TABLE

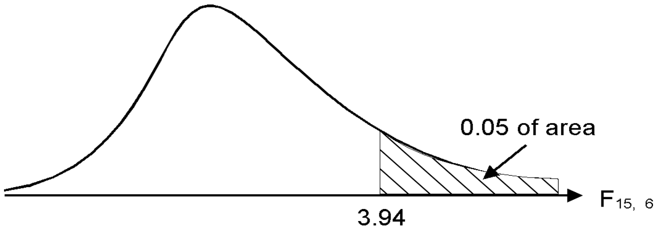
Note: If ν, the number of degrees of freedom, is greater than 30, we can approximate χ^2_{α} ,the chi-square value leaving of the area in the right tail, by

$$\chi^2_{\alpha} = \nu\left(1 - \frac{2}{9\nu} + z_{\alpha}\sqrt{\frac{2}{9\nu}}\right)^3$$

where z_{α} is the standard normal value that leaves α of the area in the right tail.

Area in right tail					Degrees of freedom
0.20	0.10	0.05	0.025	0.01	
1.642	2.706	3.841	5.024	6.635	1
3.219	4.605	5.991	7.378	9.210	2
4.642	6.251	7.815	9.348	11.345	3
5.989	7.779	9.488	11.143	13.277	4
7.289	9.236	11.070	12.833	15.086	5
8.558	10.645	12.592	14.449	16.812	6
9.803	12.017	14.067	16.013	18.475	7
11.030	13.362	15.507	17.535	20.090	8
12.242	14.684	16.919	19.023	21.666	9
13.442	15.987	18.307	20.483	23.209	10
14.631	17.275	19.675	21.920	24.725	11
15.812	18.549	21.026	23.337	26.217	12
16.985	19.812	22.362	24.736	27.688	13
18.151	21.064	23.685	26.119	29.141	14
19.311	22.307	24.996	27.488	30.578	15
20.465	23.542	26.296	28.845	32.000	16
21.615	24.769	27.587	30.191	33.409	17
22.760	25.989	28.869	31.526	34.805	18
23.900	27.204	30.144	32.852	36.191	19
25.038	28.412	31.410	34.170	37.566	20
26.171	29.615	32.671	35.479	38.932	21
27.301	30.813	33.924	36.781	40.289	22
28.429	32.007	35.172	38.076	41.638	23
29.553	33.196	36.415	39.364	42.980	24
30.675	34.382	37.652	40.647	44.314	25
31.795	35.563	38.885	41.923	45.642	26
32.912	36.741	40.113	43.194	46.963	27
34.027	37.916	41.337	44.461	48.278	28
35.139	39.087	42.557	45.722	49.588	29
36.250	40.256	43.773	46.979	50.892	30

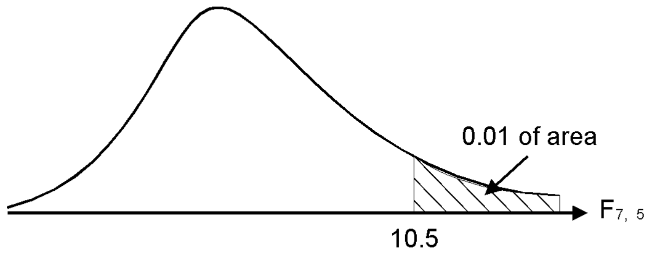
F DISTRIBUTION TABLE



Degrees of Freedom for Numerator

Degrees of Freedom for Denominator	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161	200	216	225	230	234	237	239	241	242	244	246	248	249	250	251	252	253	254
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.91	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

F DISTRIBUTION TABLE



	Degrees of Freedom for Numerator																		
Degrees of Freedom for Denominator	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	4,052	5,000	5,403	5,625	5,764	5,859	5,928	5,982	6,023	6,056	6,106	6,157	6,209	6,235	6,261	6,287	6,313	6,339	6,366
2	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.5	99.5	99.5	99.5	99.5	99.5
3	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	26.9	26.7	26.6	26.5	26.4	26.3	26.2	26.1
4	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.5	14.4	14.2	14.0	13.9	13.8	13.7	13.7	13.6
5	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
8	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
9	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
10	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17
14	8.86	6.51	5.56	5.04	4.70	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.27	3.09	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75
17	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
19	8.19	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
25	7.77	5.57	4.68	4.18	3.86	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.53	2.45	2.36	2.27	2.17
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00

CONTROL CHART FACTORS TABLE

	Factors for $\bar{x}$ Charts			Factors for R Charts	
Sample Size, n	$d_2 = \frac{R}{\sigma}$	$A_2 = \frac{3}{d_2\sqrt{n}}$	$d_3 = \frac{\sigma_R}{\sigma}$	$D_3 = 1 - \frac{3d_3}{d_2}$	$D_4 = 1 + \frac{3d_3}{d_2}$
2	1.128	1.881	0.853	0	3.269
3	1.693	0.023	0.888	0	2.574
4	2.059	0.729	0.880	0	2.282
5	2.326	0.577	0.864	0	2.114
6	2.534	0.483	0.848	0	2.004
7	2,704	0.419	0.833	0.076	1.924
8	2.847	0.373	0.820	0.136	1.864
9	2.970	0.337	0.808	0.184	1.186
10	3.078	0.308	0.797	0.223	1.777
11	3.173	0.285	0.787	0.256	1.744
12	3.258	0.266	0.779	0.283	1.717
13	3.336	0.249	0.770	0.308	1.692
14	3.407	0.235	0.763	0.328	1.672
15	3.472	0.223	0.756	0.347	1.637
16	3.532	0.212	0.750	0.363	1.637
17	3.588	0.203	0.744	0.378	1.622
18	3.640	0.194	0.739	0.391	1.609
19	3,689	0.187	0.734	0.403	1.597
20	3.735	0.180	0.729	0.414	1.586
21	3,778	0.173	0.724	0.425	1.575
22	3.819	0.167	0.720	0.434	1.566
23	3.858	0.162	0.716	0.443	1.557
24	3.895	0.157	0.712	0.452	1.548
25	3.931	0.153	0.708	0.460	1.540

Note: If $1 - 3d_3/d_2 < 0$, then $D_3 = 0$.

TABLE FOR VALUE OF CALL OPTION AS PERCENTAGE OF SHARE PRICE

Share Price Divided by PV (Exercise Price)

Standard Deviation Times Square Root of Time	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.82	0.84	0.86	0.88	0.90	0.92	0.94	0.96	0.98	1.00
0.05	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.3	0.6	1.2	2.0
0.10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.2	0.3	0.5	0.8	1.2	1.7	2.3	3.1	4.0
0.15	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.2	0.5	0.7	1.0	1.3	1.7	2.2	2.8	3.5	4.2	5.1	6.0
0.20	0.0	0.0	0.0	0.0	0.0	0.1	0.4	0.8	1.5	1.9	2.3	2.8	3.4	4.0	4.7	5.4	6.2	7.1	8.0
0.25	0.0	0.0	0.0	0.1	0.2	0.5	1.0	1.8	2.8	3.3	3.9	4.5	5.2	5.9	6.6	7.4	8.2	9.1	9.9
0.30	0.0	0.1	0.1	0.3	0.7	1.2	2.0	3.1	4.4	5.0	5.7	6.3	7.0	7.8	8.6	9.4	10.2	11.1	11.9
0.35	0.1	0.2	0.4	0.8	1.4	2.3	3.3	4.6	6.2	6.8	7.5	8.2	9.0	9.8	10.6	11.4	12.2	13.0	13.9
0.40	0.2	0.5	0.9	1.6	2.4	3.5	4.8	6.3	8.0	8.7	9.4	10.2	11.0	11.7	12.5	13.4	14.2	15.0	15.9
0.45	0.5	1.0	1.7	2.6	3.7	5.0	6.5	8.1	9.9	10.6	11.4	12.2	12.9	13.7	14.5	15.3	16.2	17.0	17.8
0.50	1.0	1.7	2.6	3.7	5.1	6.6	8.2	10.0	11.8	12.6	13.4	14.2	14.9	15.7	16.5	17.3	18.1	18.9	19.7
0.55	1.7	2.6	3.8	5.1	6.6	8.3	10.0	11.9	13.8	14.6	15.4	16.1	16.9	17.7	18.5	19.3	20.1	20.9	21.7
0.60	2.5	3.7	5.1	6.6	8.3	10.1	11.9	13.8	15.8	16.6	17.4	18.1	18.9	19.7	20.5	21.3	22.0	22.8	23.6
0.65	3.6	4.9	6.5	8.2	10.0	11.9	13.8	15.8	17.8	18.6	19.3	20.1	20.9	21.7	22.5	23.2	24.0	24.7	25.5
0.70	4.7	6.3	8.1	9.9	11.9	13.8	15.8	17.8	19.8	20.6	21.3	22.1	22.9	23.6	24.4	25.2	25.9	26.6	27.4
0.75	6.1	7.9	9.8	11.7	13.7	15.8	17.8	19.8	21.8	22.5	23.3	24.1	24.8	25.6	26.3	27.1	27.8	28.5	29.2
0.80	7.5	9.5	11.5	13.6	15.7	17.7	19.8	21.8	23.7	24.5	25.3	26.0	26.8	27.5	28.3	29.0	29.7	30.4	31.1
0.85	9.1	11.2	13.3	15.5	17.6	19.7	21.8	23.8	25.7	26.5	27.2	28.0	28.7	29.4	30.2	30.9	31.6	32.2	32.9
0.90	10.7	13.0	15.2	17.4	19.6	21.7	23.8	25.8	27.7	28.4	29.2	29.9	30.6	31.8	32.0	32.7	33.4	34.1	34.7
0.95	12.5	14.8	17.1	19.4	21.6	23.7	25.7	27.7	29.6	30.4	31.1	31.8	32.5	33.2	33.9	34.6	35.2	35.9	36.5
1.00	14.3	16.7	19.1	21.4	23.6	25.7	27.7	29.7	31.6	32.3	33.0	33.7	34.4	35.1	35.7	36.4	37.0	37.7	38.3
1.05	16.1	18.6	21.0	23.3	25.6	27.7	29.7	31.6	33.5	34.2	34.9	35.6	36.2	36.9	37.6	38.2	38.8	39.4	40.0
1.10	18.0	20.6	23.0	25.3	27.5	29.6	31.6	33.5	35.4	36.1	36.7	37.4	38.1	38.7	39.3	40.0	40.6	41.2	41.6
1.15	20.0	22.5	25.0	27.3	29.5	31.6	33.6	35.4	37.2	37.9	38.6	39.2	39.9	40.5	41.1	41.7	42.3	42.9	43.5
1.20	21.9	24.5	27.0	29.3	31.5	33.6	35.5	37.3	39.1	39.7	40.4	41.0	41.7	42.3	42.9	43.5	44.0	44.6	45.1
1.25	23.9	26.5	29.0	31.3	33.5	35.5	37.4	39.2	40.9	41.5	42.2	42.8	43.4	44.0	44.6	45.2	45.7	46.3	46.6
1.30	25.9	28.5	31.0	33.3	35.4	37.4	39.3	41.0	42.7	43.3	43.9	44.5	45.1	45.7	46.3	46.8	47.4	47.9	48.4
1.35	27.9	30.5	33.0	35.2	37.3	39.3	41.1	42.8	44.4	45.1	45.7	46.3	46.8	47.4	47.9	48.5	49.0	49.5	50.0
1.40	29.9	32.5	34.9	37.1	39.2	41.1	42.9	44.6	46.2	46.8	47.4	47.9	48.5	49.0	49.6	50.1	50.6	51.1	51.6
1.45	31.9	34.5	36.9	39.1	41.1	43.0	44.7	46.4	47.9	48.5	49.0	49.6	50.1	50.7	51.2	51.7	52.2	52.7	53.2
1.50	33.8	36.4	38.8	40.9	42.9	44.8	45.5	48.1	49.6	50.1	50.7	51.2	51.8	52.3	52.8	53.3	53.7	54.2	54.7
1.55	35.8	38.4	40.7	42.8	44.8	46.6	48.2	49.8	51.2	51.8	52.3	52.8	53.3	53.8	54.3	54.8	55.3	55.7	56.2
1.60	37.8	40.3	42.6	44.6	46.5	48.3	49.9	51.4	52.8	53.4	53.9	54.4	54.9	55.4	55.9	56.3	56.8	57.2	57.6
1.65	39.7	42.2	44.4	46.4	48.3	50.0	51.6	53.1	54.4	54.9	55.4	55.9	56.4	56.9	57.3	57.8	58.2	58.6	59.1
1.70	41.6	44.0	46.2	48.2	50.0	51.7	53.2	54.7	56.0	56.5	57.0	57.5	57.9	58.4	58.8	59.2	59.7	60.1	60.5
1.75	43.5	45.9	48.0	50.0	51.7	53.4	54.8	56.2	57.5	58.0	58.5	58.9	59.4	59.8	60.2	60.7	61.1	61.5	61.8
2.00	52.5	54.6	56.5	58.2	59.7	61.1	62.4	63.6	64.6	65.0	65.4	65.8	66.2	66.6	66.9	67.3	67.6	67.9	68.3
2.25	60.7	62.5	64.1	65.6	66.8	68.0	69.1	70.0	70.9	71.3	71.6	71.9	72.2	72.5	72.8	73.1	73.4	73.7	73.9
2.50	67.9	69.4	70.8	72.0	73.1	74.0	74.9	75.7	76.4	76.7	77.0	77.2	77.5	77.7	78.0	78.2	78.4	78.7	78.9
2.75	74.2	75.4	76.6	77.5	78.4	79.2	79.9	80.5	81.1	81.4	81.6	81.8	82.0	82.2	82.4	82.6	82.7	82.9	83.1
3.00	79.5	80.5	81.4	82.2	82.9	83.5	84.1	84.6	85.1	85.3	85.4	85.6	85.8	85.9	86.1	86.2	86.4	86.5	86.6
3.50	87.6	88.3	88.8	89.3	89.7	90.1	90.5	90.8	91.1	91.2	91.3	91.4	91.5	91.6	91.6	91.7	91.8	91.9	92.0
4.00	92.9	93.3	93.6	93.9	94.2	94.4	94.6	94.8	94.9	95.0	95.0	95.1	95.2	95.2	95.3	95.3	95.4	95.4	95.4
4.50	96.2	96.4	96.6	96.7	96.9	97.0	97.1	97.2	97.3	97.3	97.3	97.4	97.4	97.4	97.5	97.5	97.5	97.5	97.6
5.00	98.1	98.2	98.3	98.3	98.4	98.5	98.5	98.6	98.6	98.6	98.6	98.7	98.7	98.7	98.7	98.7	98.7	98.7	98.8

Share Price Divided by PV (Exercise Price)

Standard Deviation Times Square Root of Time	1.02	1.04	1.06	1.08	1.10	1.12	1.14	1.16	1.18	1.20	1.25	1.30	1.35	1.40	1.45	1.50	1.75	2.00	2.50
0.05	3.1	4.5	6.0	7.5	9.1	10.7	12.3	13.8	15.3	16.7	20.0	23.1	25.9	28.6	31.0	33.3	42.9	50.0	60.0
0.10	5.0	6.1	7.3	8.6	10.0	11.3	12.7	14.1	15.4	16.8	20.0	23.1	25.9	28.6	31.0	33.3	42.9	50.0	60.0
0.15	7.0	8.0	9.1	10.2	11.4	12.6	13.8	15.0	16.2	17.4	20.4	23.3	26.0	28.6	31.1	33.3	42.9	50.0	60.0
0.20	8.9	9.9	10.9	11.9	13.0	14.1	15.2	16.3	17.4	18.5	21.2	23.9	26.4	28.9	31.2	33.5	42.9	50.0	60.0
0.25	10.9	11.8	12.8	13.7	14.7	15.7	16.7	17.7	18.7	19.8	22.3	24.7	27.1	29.4	31.7	33.8	42.9	50.0	60.0
0.30	12.8	13.7	14.6	15.6	16.5	17.4	18.4	19.3	20.3	21.2	23.5	25.8	28.1	30.2	32.3	34.3	43.1	50.1	60.0
0.35	14.8	15.6	16.5	17.4	18.3	19.2	20.1	21.0	21.9	22.7	24.9	27.1	29.2	31.2	33.2	35.1	43.5	50.2	60.0
0.40	16.7	17.5	18.4	19.2	20.1	20.9	21.8	22.6	23.5	24.3	26.4	28.4	30.4	32.3	34.2	36.0	44.0	50.5	60.1
0.45	18.6	19.4	20.3	21.1	21.9	22.7	23.5	24.3	25.1	25.9	27.9	29.8	31.7	33.5	35.3	37.0	44.6	50.8	60.2
0.50	20.5	21.3	22.1	22.9	23.7	24.5	25.3	26.1	26.8	27.6	29.5	31.3	33.1	34.8	36.4	38.1	45.3	51.3	60.4
0.55	22.4	23.2	24.0	24.8	25.5	26.3	27.0	27.8	28.5	29.2	31.0	32.8	34.5	36.1	37.7	39.2	46.1	51.9	60.7
0.60	24.3	25.1	25.8	26.6	27.3	28.1	28.8	29.5	30.2	30.9	32.6	34.3	35.9	37.5	39.0	40.4	47.0	52.5	61.0
0.65	26.2	27.0	27.7	28.4	29.1	29.8	30.5	31.2	31.9	32.6	34.2	35.8	37.4	38.9	40.3	41.7	48.0	53.3	61.4
0.70	28.1	28.8	29.5	30.2	30.9	31.6	32.3	32.9	33.6	34.2	35.8	37.3	38.8	40.3	41.6	43.0	49.0	54.0	61.9
0.75	29.9	30.6	31.3	32.0	32.7	33.3	34.0	34.6	35.3	35.9	37.4	38.9	40.3	41.7	43.0	44.3	50.0	54.9	62.4
0.80	31.8	32.4	33.1	33.8	34.4	35.1	35.7	36.3	36.9	37.5	39.0	40.4	41.8	43.1	44.4	45.6	51.1	55.8	63.0
0.85	33.6	34.2	34.9	35.5	36.2	36.8	37.4	38.0	38.6	39.2	40.6	41.9	43.3	44.5	45.8	46.9	52.2	56.7	63.6
0.90	35.4	36.0	36.6	37.3	37.9	38.5	39.1	39.6	40.2	40.8	42.1	43.5	44.7	46.0	47.1	48.3	53.3	57.6	64.3
0.95	37.2	37.8	38.4	39.0	39.6	40.1	40.7	41.3	41.8	42.4	43.7	45.0	46.2	47.4	48.5	49.6	54.5	58.6	65.0
1.00	38.9	39.5	40.1	40.7	41.2	41.8	42.4	42.9	43.4	44.0	45.2	46.5	47.6	48.8	49.9	50.9	55.6	59.5	65.7
1.05	40.6	41.2	41.8	42.4	42.9	43.5	44.0	44.5	45.0	45.5	46.8	48.0	49.1	50.2	51.2	52.2	56.7	60.5	66.5
1.10	42.3	42.9	43.5	44.0	44.5	45.1	45.6	46.1	46.6	47.1	48.3	49.4	50.5	51.6	52.6	53.5	57.9	61.5	67.2
1.15	44.0	44.6	45.1	45.6	46.2	46.7	47.2	47.7	48.2	48.6	49.8	50.9	51.9	52.9	53.9	54.9	59.0	62.5	68.0
1.20	45.7	46.2	46.7	47.3	47.8	48.3	48.7	49.2	49.7	50.1	51.3	52.3	53.3	54.3	55.2	56.1	60.2	63.5	68.8
1.25	47.3	47.8	48.4	48.8	49.3	49.8	50.3	50.7	51.2	51.6	52.7	53.7	54.7	55.7	56.6	57.4	61.3	64.5	69.6
1.30	48.9	49.4	49.9	50.4	50.9	51.3	51.8	52.2	52.7	53.1	54.1	55.1	56.1	57.0	57.9	58.7	62.4	65.5	70.4
1.35	50.5	51.0	51.5	52.0	52.4	52.9	53.3	53.7	54.1	54.6	55.6	56.5	57.4	58.3	59.1	59.9	63.5	66.5	71.1
1.40	52.1	52.6	53.0	53.5	53.9	54.3	54.8	55.2	55.6	56.0	56.9	57.9	58.7	59.6	60.4	61.2	64.6	67.5	71.9
1.45	53.6	54.1	54.5	55.0	55.4	55.8	56.2	56.6	57.0	57.4	58.3	59.2	60.0	60.9	61.6	62.4	65.7	68.4	72.7
1.50	55.1	55.6	56.0	56.4	56.8	57.2	57.6	58.0	58.4	58.8	59.7	60.5	61.3	62.1	62.9	63.6	66.8	69.4	73.5
1.55	56.6	57.0	57.4	57.8	58.2	58.6	59.0	59.4	59.7	60.1	61.0	61.8	62.6	63.3	64.1	64.7	67.8	70.3	74.3
1.60	58.0	58.5	58.9	59.2	59.6	60.0	60.4	60.7	61.1	61.4	62.3	63.1	63.8	64.5	65.2	65.9	68.8	71.3	75.1
1.65	59.5	59.9	60.2	60.6	61.0	61.4	61.7	62.1	62.4	62.7	63.5	64.3	65.0	65.7	66.4	67.0	69.9	72.2	75.9
1.70	60.9	61.2	61.6	62.0	62.3	62.7	63.0	63.4	63.7	64.0	64.8	65.5	66.2	66.9	67.5	68.2	70.9	73.1	76.6
1.75	62.2	62.6	62.9	63.3	63.6	64.0	64.3	64.6	64.9	65.3	66.0	66.7	67.4	68.0	68.7	69.2	71.9	74.0	77.4
2.00	68.6	68.9	69.2	69.5	69.8	70.0	70.3	70.6	70.8	71.1	71.7	72.3	72.9	73.4	73.9	74.4	76.5	78.3	81.0
2.25	74.2	74.4	74.7	74.9	75.2	75.4	75.6	75.8	76.0	76.3	76.8	77.2	77.7	78.1	78.5	78.9	80.6	82.1	84.3
2.50	79.1	79.3	79.5	79.7	79.9	80.0	80.2	80.4	80.6	80.7	81.1	81.5	81.9	82.2	82.6	82.9	84.3	85.4	87.2
2.75	83.3	83.4	83.6	83.7	83.9	84.0	84.2	84.3	84.4	84.6	84.9	85.2	85.5	85.8	86.0	86.3	87.4	88.3	89.7
3.00	86.8	86.9	87.0	87.1	87.3	87.4	87.5	87.6	87.7	87.8	88.1	88.3	88.5	88.8	89.0	89.2	90.0	90.7	91.8
3.50	92.1	92.1	92.2	92.3	92.4	92.4	92.5	92.6	92.6	92.7	92.8	93.0	93.1	93.3	93.4	93.5	94.0	94.4	95.1
4.00	95.5	95.5	95.6	95.6	95.7	95.7	95.7	95.8	95.8	95.8	95.9	96.0	96.1	96.2	96.2	96.3	96.6	96.8	97.2
4.50	97.6	97.6	97.6	97.6	97.7	97.7	97.7	97.7	97.8	97.8	97.8	97.9	97.9	97.9	98.0	98.0	98.2	98.3	98.5
5.00	98.8	98.8	98.8	98.8	98.8	98.8	98.8	98.8	98.9	98.9	98.9	98.9	98.9	99.0	99.0	99.0	99.1	99.1	99.2

Table for N(x) When x ≤ 0

This table shows values of N(x) for x ≤ 0. The table should be used with interpolation. For example,

$$\begin{aligned} N(-0.1234) &= N(-0.12) - 0.34[N(-0.12) - N(-0.13)] \\ &= 0.4522 - 0.34 \times (0.4522 - 0.4483) \\ &= 0.4509 \end{aligned}$$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
–0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
–0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
–0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
–0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
–0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
–0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
–0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
–0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
–0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
–0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
–1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
–1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
–1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
–1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
–1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
–1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
–1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
–1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
–1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
–1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
–2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
–2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
–2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
–2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
–2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
–2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
–2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
–2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
–2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
–2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
–3.0	0.0014	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
–3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
–3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
–3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
–3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
–3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
–3.6	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
–3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
–3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
–3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
–4.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table for N(x) When x ≥ 0

This table shows values of N(x) for x ≥ 0. The table should be used with interpolation. For example,

$$\begin{aligned} N(0.6278) &= N(0.62) + 0.78[N(0.63) - N(0.62)] \\ &= 0.7324 + 0.78 \times (0.7357 - 0.7324) \\ &= 0.7350 \end{aligned}$$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9986	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE FOR RELATIONSHIP BETWEEN NOMINAL AND EFFECTIVE RATES OF INTEREST AND DISCOUNT

$i^{(p)} = [(1 + i)^{1/p} - 1]p$ $\quad$ $d^{(p)} = [1 - (1 - d)^{1/p}]p$

$$d = \frac{i}{1+i}$$

Interest \ Effective rate	$i^{(2)}$	$i^{(4)}$	$i^{(12)}$	d	$d^{(2)}$	$d^{(4)}$	$d^{(12)}$
0.01	0.0100	0.0100	0.0100	0.0099	0.0099	0.0099	0.0099
0.02	0.0199	0.0199	0.0198	0.0196	0.0197	0.0198	0.0198
0.03	0.0298	0.0297	0.0296	0.0291	0.0293	0.0294	0.0295
0.04	0.0396	0.0394	0.0393	0.0385	0.0388	0.0390	0.0392
0.05	0.0494	0.0491	0.0489	0.0476	0.0482	0.0485	0.0487
0.06	0.0591	0.0587	0.0584	0.0566	0.0574	0.0578	0.0581
0.07	0.0688	0.0682	0.0678	0.0654	0.0665	0.0671	0.0675
0.08	0.0785	0.0777	0.0772	0.0741	0.0755	0.0762	0.0767
0.09	0.0881	0.0871	0.0865	0.0826	0.0843	0.0853	0.0859
0.10	0.0976	0.0965	0.0957	0.0909	0.0931	0.0942	0.0949
0.11	0.1071	0.1057	0.1048	0.0991	0.1017	0.1030	0.1039
0.12	0.1166	0.1149	0.1139	0.1071	0.1102	0.1117	0.1128
0.13	0.1260	0.1241	0.1228	0.1150	0.1186	0.1204	0.1216
0.14	0.1354	0.1332	0.1317	0.1228	0.1268	0.1289	0.1303
0.15	0.1448	0.1422	0.1406	0.1304	0.1350	0.1373	0.1390
0.16	0.1541	0.1512	0.1493	0.1379	0.1430	0.1457	0.1475
0.17	0.1633	0.1601	0.1580	0.1453	0.1510	0.1540	0.1560
0.18	0.1726	0.1690	0.1667	0.1525	0.1589	0.1621	0.1644
0.19	0.1817	0.1778	0.1752	0.1597	0.1666	0.1702	0.1727
0.20	0.1909	0.1865	0.1837	0.1667	0.1743	0.1782	0.1809
0.21	0.2000	0.1952	0.1921	0.1736	0.1818	0.1861	0.1891
0.22	0.2091	0.2039	0.2005	0.1803	0.1893	0.1940	0.1972
0.23	0.2181	0.2125	0.2088	0.1870	0.1967	0.2017	0.2052
0.24	0.2271	0.2210	0.2171	0.1935	0.2039	0.2094	0.2132
0.26	0.2450	0.2379	0.2334	0.2063	0.2183	0.2246	0.2289
0.28	0.2627	0.2546	0.2494	0.2188	0.2322	0.2394	0.2443
0.30	0.2804	0.2712	0.2653	0.2308	0.2459	0.2539	0.2595
0.32	0.2978	0.2875	0.2809	0.2424	0.2592	0.2682	0.2744
0.34	0.3152	0.3036	0.2963	0.2537	0.2723	0.2822	0.2891
0.36	0.3324	0.3196	0.3115	0.2647	0.2850	0.2960	0.3036
0.38	0.3495	0.3354	0.3264	0.2754	0.2975	0.3095	0.3178
0.40	0.3664	0.3510	0.3412	0.2857	0.3097	0.3227	0.3318

TABLE FOR RELATIONSHIP BETWEEN NOMINAL AND EFFECTIVE RATES OF INTEREST AND DISCOUNT

I/effective rate / Interest	$i/i^{(2)}$	$i/i^{(4)}$	$i/i^{(12)}$	$i/d^{(2)}$	$i/d^{(4)}$	$i/d^{(12)}$
0.01	1.0025	1.0037	1.0046	1.0075	1.0062	1.0054
0.02	1.0050	1.0075	1.0091	1.0150	1.0125	1.0108
0.03	1.0074	1.0112	1.0137	1.0224	1.0187	1.0162
0.04	1.0099	1.0149	1.0182	1.0299	1.0249	1.0215
0.05	1.0123	1.0186	1.0227	1.0373	1.0311	1.0269
0.06	1.0148	1.0222	1.0272	1.0448	1.0322	1.0372
0.07	1.0172	1.0259	1.0317	1.0522	1.0434	1.0375
0.08	1.0196	1.0295	1.0362	1.0596	1.0495	1.0428
0.09	1.0220	1.0331	1.0406	1.0670	1.0556	1.0481
0.10	1.0244	1.0368	1.0450	1.0744	1.0618	1.0534
0.11	1.0268	1.0404	1.0495	1.0818	1.0679	1.0586
0.12	1.0292	1.0439	1.0539	1.0892	1.0739	1.0639
0.13	1.0315	1.0475	1.0583	1.0965	1.0800	1.0691
0.14	1.0339	1.0511	1.0626	1.1039	1.0861	1.0743
0.15	1.0362	1.0546	1.0670	1.1112	1.0921	1.0795
0.16	1.0385	1.0581	1.0714	1.1185	1.0981	1.0847
0.17	1.0408	1.0617	1.0757	1.1258	1.1042	1.0899
0.18	1.0431	1.0652	1.0800	1.1331	1.1102	1.0950
0.19	1.0454	1.0687	1.0843	1.1404	1.1162	1.1002
0.20	1.0477	1.0722	1.0887	1.1477	1.1222	1.1053
0.21	1.0500	1.0756	1.0929	1.1550	1.1281	1.1104
0.22	1.0523	1.0791	1.0972	1.1623	1.1341	1.1155
0.23	1.0545	1.0825	1.1015	1.1695	1.1400	1.1206
0.24	1.0568	1.0860	1.1057	1.1768	1.1460	1.1257
0.26	1.0612	1.0928	1.1142	1.1912	1.1578	1.1359
0.28	1.0657	1.0996	1.1226	1.2057	1.1696	1.1460
0.30	1.0701	1.1064	1.1310	1.2201	1.1814	1.1560
0.32	1.0745	1.1131	1.1393	1.2345	1.1931	1.1660
0.34	1.0788	1.1197	1.1476	1.2488	1.2047	1.1759
0.36	1.0831	1.1264	1.1559	1.2631	1.2164	1.1859
0.38	1.0874	1.1330	1.1641	1.2774	1.2280	1.1957
0.40	1.0916	1.1395	1.1722	1.2916	1.2395	1.2055

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